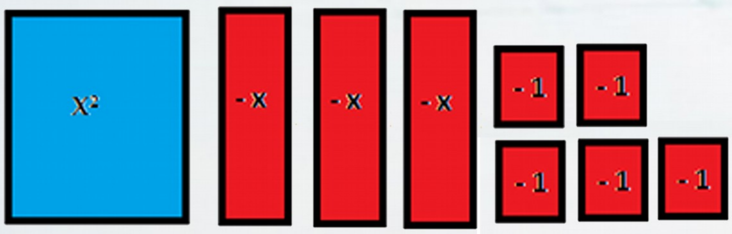
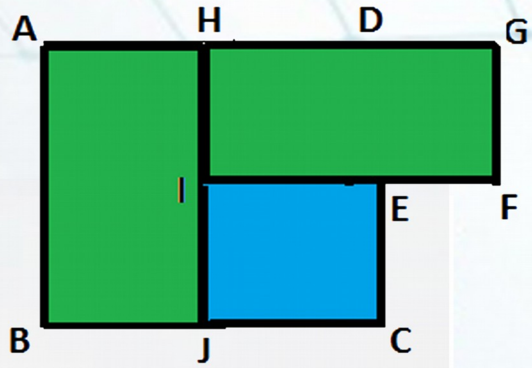
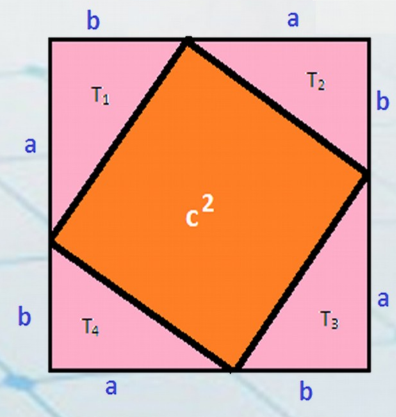
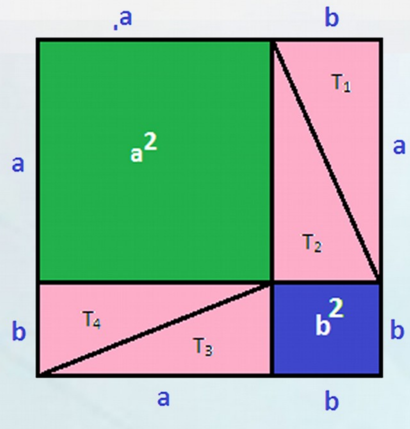
Manual for Teaching Learning Materials (TLMs) in Mathematics at the Upper Primary Level









DIRECTORATE OF EDUCATIONAL RESEARCH & TRAINING,

NONGRIMMAW, LAITUMKHRAH, SHILLONG - 793011

MEGHALAYA

2017

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Foreword

Mathematics is one of the most important subjects in the school curriculum. It helps in problem-solving and decision-making through its applications to real life situation in familiar as well as non-familiar contexts. According to the National Curriculum Framework (NCF) 2005, developing children's abilities for mathematisation is the main goal of mathematics education. The Position Paper on Teaching of Mathematics (NCF- 2005) mentioned some problems in teaching and learning of mathematics. Two of these problems are:

- A majority of children have a sense of fear and failure regarding Mathematics. Hence, they give up early on, and drop out of serious mathematical thinking.
- Lack of teacher preparation and support in the teaching of Mathematics.

One of its recommendations is to enrich teachers with a variety of mathematical resources. There are many mechanisms that need to be ensured to offer better teacher support and professional development, but the essential requirement is that of a large treasury of resource material which teachers can access freely as well as contribute to. It also observes that school mathematics must be activity-oriented.

This Manual for Teaching Learning Materials (TLMs) in Mathematics at the Upper Primary Level has been developed by the Directorate of Educational Research & Training, Shillong, Meghalaya in order to help teachers make teaching and learning of Mathematics at the upper primary level interesting, joyful and effective. I hope that it will make a significant contribution for improving the teaching and learning of Mathematics at the upper primary level.

I express my gratitude to Mr. K. Dympep, Selection Grade Lecturer, DIET Sohra, Mr. Wallambor Wanswett, Selection Grade Lecturer, DIET Thadlaskein and Mr. S. Kharchandy, Assistant Professor, Shillong College for their keen interest and support in the development of this Manual. I am also grateful to the teachers participating in the development workshop for their contributions.

I sincerely acknowledge and appreciate the initiative and hard work done by Mr. P. B. Lartang, Selection Grade Lecturer, DERT, Shillong (Co-ordinator for Development of this Manual) who contributed to the development and finalization of this Manual.

I would also like to acknowledge and appreciate the hard work done by Ms. Adella Lyngdoh in drawing the figures.

We look forward for comments and suggestions for further improvement and refinement of this Manual.

(T. G. Momin)
Director
Directorate of Educational Research & Training
Meghalaya, Shillong

DEVELOPMENT TEAM

Resource Persons:

Mr. P. B. Lartang, Selection Grade Lecturer, DERT, Shillong (Co-ordinator)

Mr. K. Dympep, Selection Grade Lecturer, DIET Sohra, East Khasi Hills

Mr. Wallambor Wanswett, Selection Grade Lecturer, DIET Thadlaskein, West Jaintia Hills

Mr. S. Kharchandy, Assistant Professor, Shillong College, Shillong

Participants/Teachers:

Smt. Iohsngew Dkhar, Lumjingshai Sec. School, Tluh, East Jaintia Hills

Smt. Kyllabyllin Diengdoh, Christian U.P. School, Siejlieh, Nongstoin, West Khasi Hills

Shri. Pet Dhar, Nongbah Govt. U.P. School, Nongbah, West Jaintia Hills

Shri. Firstborn Myrthong, Nongtluh U.P. School, Diwon, Ri Bhoi

Smt. Sharailin Puwein, Mainop CoG UP School, West Khasi Hills

Shri. Mohan Ch. Karfay, Gorkha Pathshala UP School, Shillong, East Khasi Hills

Shri. S. Leven Cliff Talang, Wahiajer Govt UP School, Wahiajer, West Jaintia Hills

Shri. Kynphamlang Marbaniang, Rangthong Ch Def UP School, South West Khasi Hills

Shri. Kitbokstar Lyngkhoi, Ngunraw (Def) UP School, Ngunraw, South West Khasi Hills

Shri. Reynold Basiawmoit, Gorkha Sec. School, Shillong, East Khasi Hills

Shri. B. Ryntathiang, Tirot Sing Memo. Govt. UP School, Nongkhlaw, West Khasi Hills

Shri. Welcome Dkhar, Sein Raij Branch UP School, Jalyiah, East Jaintia Hills

Smt. Daphiralyne Lyngdoh, Govt. UP School, Mawryngkneng, East Khasi Hills

Shri. Eiban Nongsiej, Myriaw Govt. UP School, West Khasi Hills

Smt. Beautiful Shylla, Ieng Swer Memorial School, Dkhiah, East Jaintia Hills

Smt. Shoma Ahmed, Akonggri UP School, Tura, West Garo Hills

Shri. Marshal Tito Suting, Warmawsaw UP School, Ri Bhoi

Smt. Sathi Choudhury, Umpling(R) Girls' UP School, Shillong, East Khasi Hills

Shri. Jeristar Nongsiej, Langtor Govt. UP School, West Khasi Hills

Shri. E. W. Dohtdong, United Christian UP Schol, Krang, East Khasi Hills

Shri. Nicholas Hahshah, Sohlaper UP School, Diengpasoh, East Khasi Hills

Smt. Aiphiral Myrboh, Riwar Jaintia CoG Sec. School, Amlarem, West Jaintia Hills

Shri. W. Manner, Nongryngkoh UP School, East Khasi Hills

Shri. Narayan Paul, Jagarani UP School, Baghmara, South Garo Hills

Shri. Balaram Chowdhury, Tikrikilla Govt. UP School, West Garo Hills

Shri Sheaffer Nalle Malngiang, Govt. UP School, Mawsynram, East Khasi Hills

Shri Ekhel Marwein, RKM UP School, Bholaganj, East Khasi Hills

Shri Joydeep Debnath, Nayabasli (SSA) UP School, East Khasi Hills

Shri Blanstar Lyngdoh, Christ Church Sec.School, Mawlai, East Khasi Hills

Shri Defendar Lamat, Umladkhur UP School (SSA), West Jaintia Hills

Shri Hesper S. Shullai, Jowai Govt. UP School, Jowai, West Jaintia Hills

Shri Alexander Rongrin, Rongrin Govt. UP School, Umdang, West Khasi Hills

Shri Edison Lyngdoh, Govt. Sr. Basic School, Mynso, West Jaintia Hills

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ACTIVITY 1

Objective: To verify that addition of whole numbers is commutative

Materials required: Chart paper, sheets of paper, scissors, ruler, glue, pen/pencil, colours.

Procedure:

1. Take a Chart Paper and cut two strips eachof length a = 5 units (say) and breadth 1 unit.
2. Divide each strip into five equal parts to form unit squares and colour them green as shown in Fig. 1 (a)

Fig. 1 (a)

3. Similarly, make two strips each containing b, say 3 unit squares and colour them pink, as shown in Fig. 1 (b)

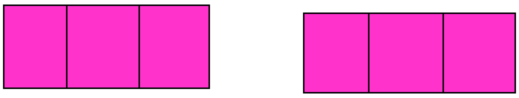


Fig. 1 (b)

4. Draw two lines l_1 and l_2 on a sheet of paper as shown in Fig. 2.

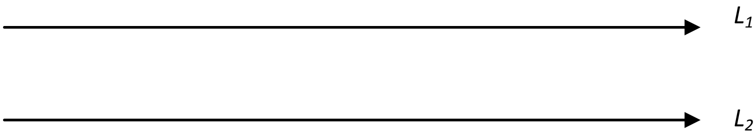


Fig.2

5. Now arrange/paste the green and pink strips side by side on lines l_1 and l_2, as shown in Fig. 3

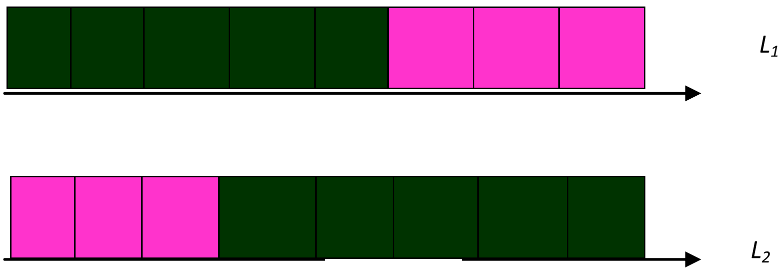


Fig.3

Observation:

1. From Fig. 3,
   a) The length of the combined strips on line l_1 = 5+3 =8
   b) The length of the combined strips on line l_2 = 3+5 =8
   c) One can see that the length of combined strips on l_1 is the same as the length of the combined strips on l_2.
   d) So, 5 + 3 = 3 + 5

      i.e., addition of 5 and 3 is commutative

2. Repeat this activity by taking different pairs of numbers like 2, 5; 4, 6; 5, 7 and strips corresponding to these pairs.

Conclusion: Addition of whole numbers is commutative, i.e., if a and b are any two whole numbers, then a + b = b + a.

ACTIVITY 2

Objective: To verify the distributive property of multiplication over addition of whole numbers.

Materials required: Chart paper or grids of different dimensions, scissors, ruler, colours,pen/pencil.

Procedure:

1. Make a grid of 5 rows by 10 columns (5 rows of 10 squares) and colour each unit square with the same colour (say green). Make a cut-out as shown in Fig. 1.

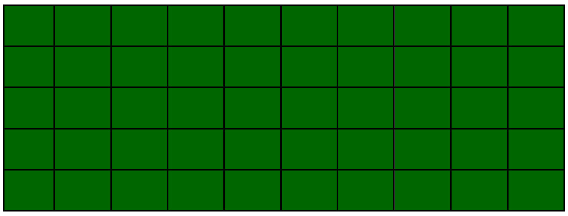


Fig. 1

2. Make a set of two grid papers, one with 5 rows by 7 columns (5 rows of 7 squares) and another with 5 rows by 3 columns (5 rows of 3 squares). Colour each unit square with the same colour (say pink) and make their cut-outs as shown in Fig. 2.

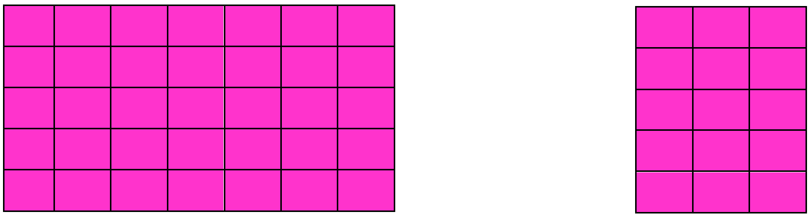


Fig. 2

3. Place the two grids of Fig. 2 over the grid in Fig. 1
4. Both the grids of Fig. 2 when arranged side by side, leaving no space between them, will exactly cover the grid in Fig.1.

Observation:

1. On actual counting of the unit squares,
   a) 5 rows of 10 squares = 5 x 10 = 50 squares
   b) 5 rows of 7 squares + 5 rows of 3 squares = 5 x 7 + 5 x 3
   = 35 + 15
   = 50 squares

c) Thus, $5 \times 10 = 5 \times 7 + 5 \times 3$

i.e., $5 \times (7 + 3) = 5 \times 7 + 5 \times 3$

2. Repeat this activity for different sets such as 10, 4, 6; 12, 4, 8; etc.

Conclusion: In general, if a, b and c are any three whole numbers, then,

$$a \times (b + c) = a \times b + a \times c$$

ACTIVITY 3

Objective: To verify the distributive property of multiplication over addition of whole numbers.

Materials required: Chart paper, ruler, pen, pencil, colours.

Procedure:

1. Construct a rectangle ABCD with sides 'a' and (b+c), and shade with green colour as shown in Fig. 1 [say a = 5cm, b = 2cm, c = 1cm]

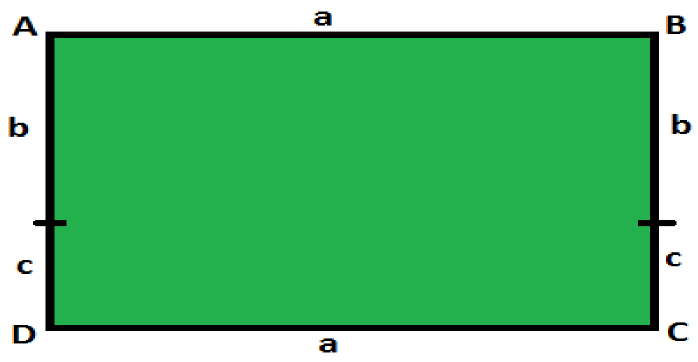


Fig. 1

2. Construct another rectangle EFGH with same sides 'a' and (b+c), as shown in Fig. 2.

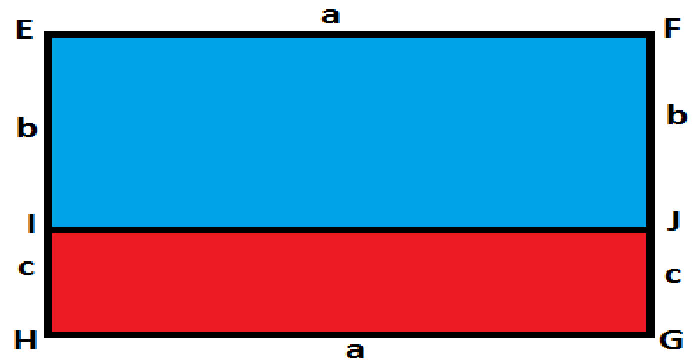


Fig. 2

3. Mark points I and J on sides EH and FG respectively such that EI = FJ=b and IH=JG = c. Join IJ (Fig.2).
4. Shade the rectangle EFJI with blue colour and the rectangle IJGH with red colour (Fig. 2)

Observation:

1. From Fig. 1, area of the rectangle ABCD = a × (b+c)
2. From Fig. 2,
   a) Area of the rectangle EFJI = a x b
   b) Area of the rectangle IJGH = a x c
3. Also, area of rectangle ABCD = area of rectangle EFGH

   i.e., area of rectangle ABCD = area of EFJI + area of IFGH

   so, $a \times (b + c) = a \times b + a \times c$

4. Repeat the activity by taking different sets of values of a,b and c.

Conclusion: If a, b and c are any whole numbers then,

$$a \times (b + c) = a \times b + a \times c$$

ACTIVITY 4

Objective: To find the L.C.M. of two numbers.

Materials required: Chart paper or white drawing sheet, scissors, knife/cutter, pen/pencil.

Procedure:

1. Make three grids each of size, say, 10 cm x 10 cm and write numbers 1 to 100 on each of them. (Fig. 1)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Fig. 1

2. From one grid, cut out the multiples of one of the numbers, say 8 (Fig. 2)

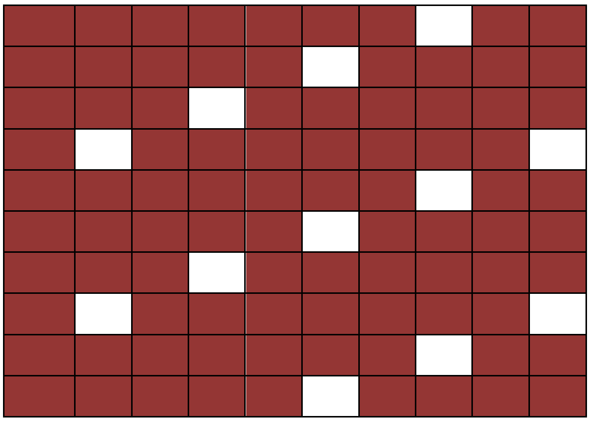


Fig. 2

3. Cut out the multiples of another number, say 12, from another grid. (Fig. 3)

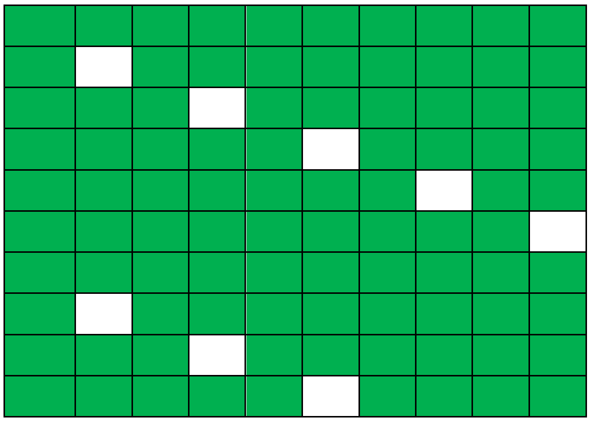


Fig. 3

4. Place the two cut-out grids one above the other over the base grid (Fig. 4). We get the common multiples of 8 and 12 visible through the holes.

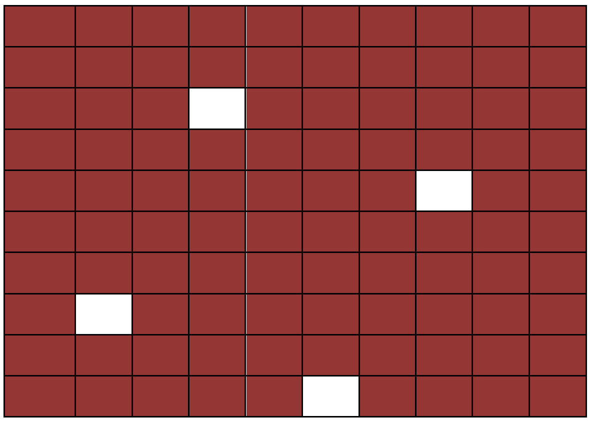


Fig. 4

5. The smallest of these common multiples is the L.C.M. of the two numbers, i.e., of 8 and 12, which is 24.

Observation:

1. Placing the cut-out grid (of Fig.2) over the base grid, the multiples of 8 (visible through the holes) are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96.
2. Placing the cut-out grid (of Fig.3) over the base grid, the multiples of 12 (visible through the holes) are 12, 24, 36, 48, 60, 72, 84, 96.
3. Placing both the cut-out grids one above the other over the base grid, the common multiples of 8 and 12 (visible through the holes) are 24, 48, 72, 96.
4. The smallest common multiple of 8 and 12 (visible through the holes) is 24.

Conclusion: The L.C.M. of 8 and 12 is 24.

ACTIVITY 5

Objective: To find fractions equivalent to a given fraction.

Materials required: Chart paper, sheet of paper, glue, ruler, pen, pencil, colours

Procedure:

Let us find fractions equivalent to $\frac{1}{2}$ by paper folding.

1. Draw four rectangles of dimensions 16 cm x 2 cm (say) on a sheet of paper and cut these out with the help of scissors.
2. Fold all the four strips into two equal parts.
3. Unfold one of them, colour one part and paste the strip on a chart paper (or another sheet of paper) as shown in Fig. 1.

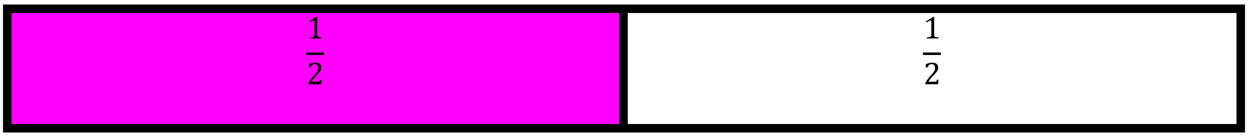


Fig. 1

4. Take the remaining three strips and fold them again.
5. Unfold one of the three strips and colour two equal parts as shown in Fig. 2.

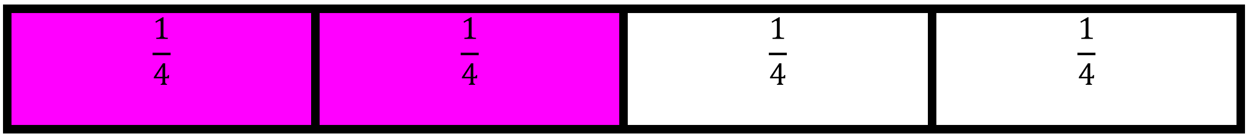


Fig. 2

6. Paste it on the chart paper/sheet of paper just below the first strip as shown in Fig. 5.
7. Take the remaining two strips and fold them again.
8. Unfold one of the two strips and colour four equal parts as shown in Fig. 3.

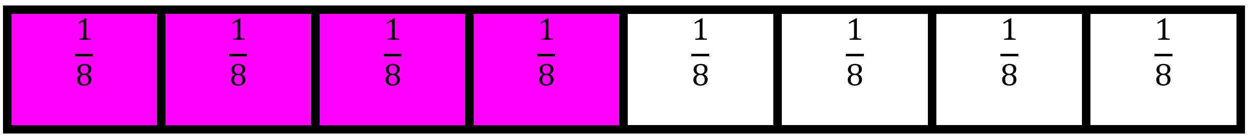


Fig. 3

9. Paste it on the chart paper/sheet of paper just below the second strip as shown in Fig. 5.
10. Take the fourth strip and fold it again once. Unfold the two strips and colour eight equal parts as shown in Fig. 4.

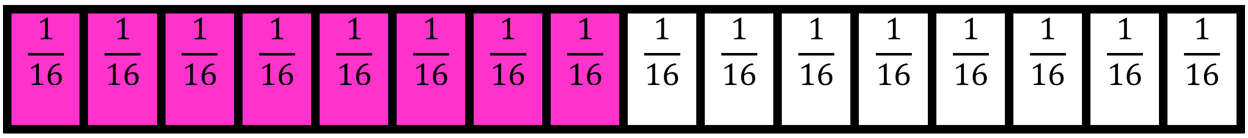


Fig. 4

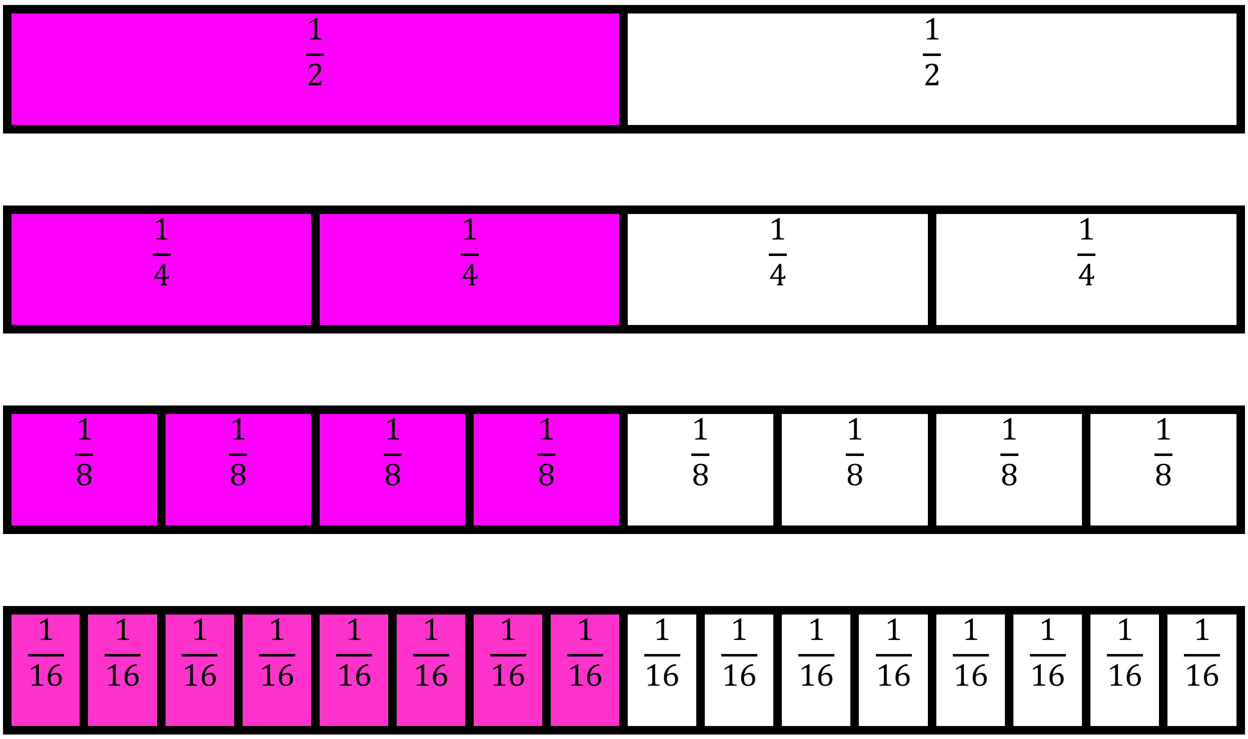


Fig. 5

Observation:

1. In all the four figures (Figs. 1 – 4), the coloured portions in all the four strips are equal. (Fig. 5)
2. (a) In Fig. 1, coloured portion represents the fraction $\frac{1}{2}$.

   (b) In Fig. 2, coloured portion represents the fraction $\frac{2}{4}$.

   (c) In Fig. 3, coloured portion represents the fraction $\frac{4}{8}$.

   (d) In Fig. 4, coloured portion represents the fraction $\frac{8}{16}$.

3. Since the coloured portions of all 4 strips are equal,

so, $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$

Conclusion: Thus, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$ are fractions equivalent to the fraction $\frac{1}{2}$.

*[**Note:***

1) In a similar way, the activity can be performed for finding equivalent fractions of $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ etc.

2) This activity can be used to explain the meaning of equivalent fractions.]

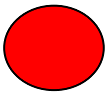
ACTIVITY 6

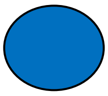
Objective: To add integers using counters (or buttons) of different colours.

Materials required: Square or circular counters coloured differently on both the faces, one face red and the other face blue.

Procedure:

1. Consider the red side of the counter as positive (+) and the blue side of the counter as negative $(-)$

   i.e.,  or  represents +1

   and  or  represents –1.

2. **Adding two positive integers**, say, (+3) and (+4).
   a) First, take 3 counters and place them in a row in such a way that the top faces are red.
   b) Now, take 4 more counters and place them in the other row (below), so that their top faces are also red (Fig. 1).

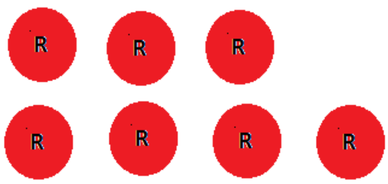


Fig. 1

   c) Since the top faces of all the counters are red, so count all these counters together.
   d) There are altogether 7 counters of red colour.
   e) So, we get the sum, (+3) + (+4) = +7.

3. **Adding two negative integers**, say, -3 and -5.
   a) First take 3 counters and place them in a row in such a way that the top faces are blue.
   b) Now, take 5 more counters and place them in the other row (below), so that their top faces are also blue (Fig. 2).

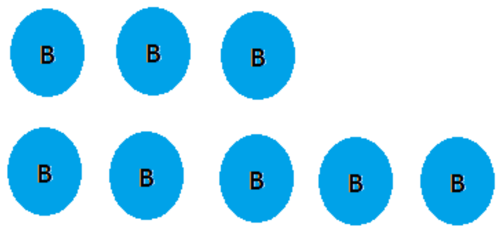


Fig. 2

c) Since the top faces of all the counters are blue, so count all these counters together.
d) There are altogether 8 counters of blue colour.
e) So, we get the sum as,$(-3) + (-5) = -8$.

4. **Adding one positive and one negative integer.**

i) Example (+5) and (−2) (when numerical value of positive integer is greater).
a) First, take 5 counters and place them in a row in such a way that their top faces are red.
b) Now, take 2 more counters and place them in the other row (below), so that their top faces are blue (Fig. 3)

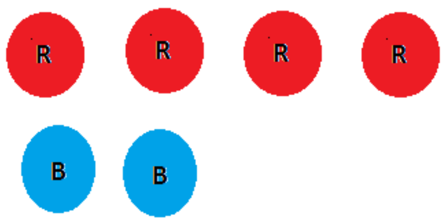


Fig.3

c) Now, match each red-faced counter with blue-faced counter by encircle one red and one blue counter as shown below in Fig. 4.

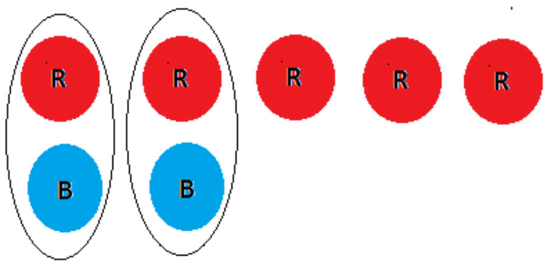


Fig. 4

d) Count the number of remaining unmatched counters and note down their colours. This will give the sum.

e) There are three red counters left. So, we get the sum as,

$$(+5) + (-2) = +3$$

ii) Similarly, adding integers like $(+2)$ and (-4) (when numerical value of negative integer is greater), there will be two blue counters left.

So, we get the sum as, $(+2) + (-4) = -2$.

Conclusion:

1. Sum of two positive integers is a positive integer.
2. Sum of two negative integers is a negative integer.
3. Sum of one positive integer and one negative integer is a:
   (i) positive integer if numerical value of positive integer is greater.
   (ii) negative integer if numerical value of negative integer is greater
4. Thus,
   (i) If the integers are of the same sign, then to find their sum, add the two integers ignoring their signs and put the sign of the two integers with the sum.
   (ii) If the integers are of the different signs, then to find their sum, subtract the smaller number from the bigger number (ignoring their signs) and put the sign of the bigger number with the sum.

Note: *You can also take red side of the counter as negative, and the blue side as positive.*

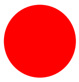
ACTIVITY 7

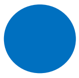
Objective: To subtract integers using counters (or buttons) of different colours.

Materials required: Square or circular counters coloured differently on both the faces/sides, say, one face red and the other face blue.

Procedure:

1. Consider the red side of the counter as positive (+) and the blue side of the counter as negative (–)

i.e.,  or  represents +1

and  or  represents –1.

2. Consider the subtraction of two positive integers, say (+2) from (+3), i.e., (+3) – (+2).
   a) First, take 3 counters and place them in a row so that their top faces are red.
   b) Take another 2 counters and place them in the other row (below) so that their top faces are also red (Fig.1).

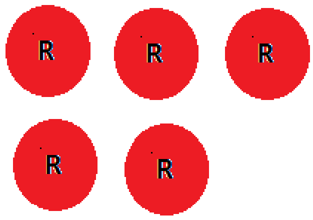


Fig.1

   c) Now, keep the counters of the first row as they are and invert the sides of the counters of the second row so that their top faces are now blue, as shown in Fig.2.

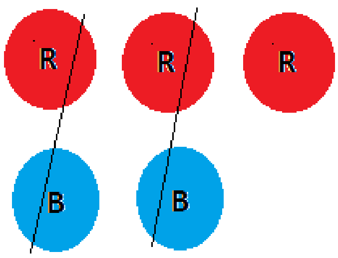


Fig.2

   d) Match red-faced counters with blue-faced counters (Fig.2).
   e) Count remaining unmatched counters and note down the colours.

f) There is one red counter left. So we get, (+3) – (+2) = +1.

3. Consider subtracting two negative integers, say, the subtraction of (–4) from (–3), i.e., (– 3) – (– 4).
   a) First take 3 counters and place them in a row so that their top faces are blue.
   b) Take another 4 counters and place them in the second row so that their top faces are also blue (Fig 3).

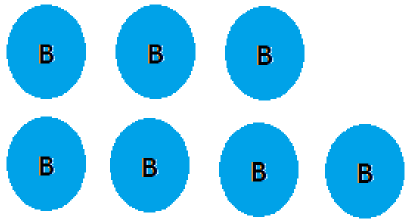


Fig.3

   c) Now keep the counters of the first row as they are and invert the sides of the counters of the second row so that their top faces are row red, as shown in Fig.4.

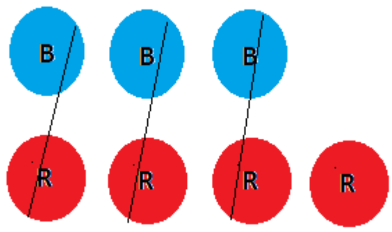


Fig.4

   d) Match blue-faced counters with red-faced counters (Fig.4).
   e) Count the remaining unmatched counters and note down the colours.
   f) There is one red counter left. So we get, (–3) – (–4) = +1.

4. Consider subtracting one positive and one negative integers, say, (+5) – (–3)
   a) First, take 5 counters and place them in a row so that their top faces are red.
   b) Take another 3 counters and place them in the second row so that their top faces are blue (Fig.5).

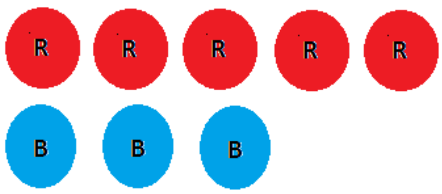


Fig.5

c) Now, keep the counters of the first row as they are and invert the sides of the counters of the second row, so that their top faces are now red, as shown in Fig.6.

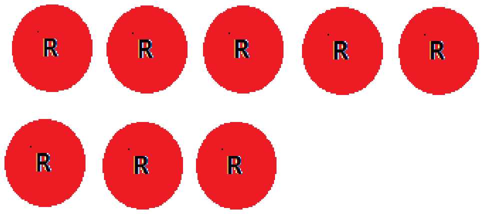


Fig.6

d) Since the top faces of all the counters are red, so count all these counters together.

e) There are altogether 8 counters of red colours. So we get, (+5) – (–3) = (+8).

Note: *You can also take red side of the counter as negative, and the blue side as positive; or use other colours.*

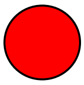
ACTIVITY 8

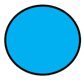
Objective: To multiply integers using countersof different colours.

Materials required: Square or circular counters coloured differently on both the faces/sides, say, one face red and the other face blue.

Procedure:

1. Denote the red side of the counter as positive (+) and the blue side of the counter as negative (–)

   i.e.,  or  represents +1

   and  or  represent -1

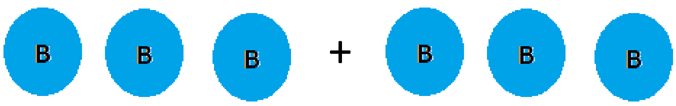
2. Multiplication of two positive integers, say, (+3) × (+2).
   a) (+3) × (+2) means you have to add (+3), two times,
      i.e., $(+3) \times (+2) = \underbrace{(+3) + (+3)}_{\text{(2 times)}} = +6.$

   b) With the help of counters,you can show this as follows:

= + 6

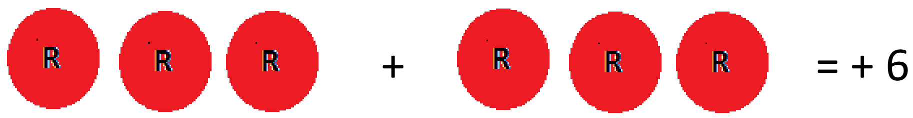
3. Multiplication of a negative integer by a positive integer, say, (–3) × (+2).
   a) (–3) × (+2) means you have to add (–3), two times,
      i.e., $(-3) \times (+2) = \underbrace{(-3) + (-3)}_{\text{(2 times)}} = -6.$

   b) With the help of counters, you can show this as follows:



= - 6

4. Multiplication of two negative integers, say, (–3) × (–2).
   a) (–3) × (–2) means add –3, (–2) times.But you cannot add a number –2 times.
   b) You know that (–2) can be written as – (+2). Therefore, (–3) × (–2) means you have add (–3), two times but from the opposite side.
   c) (–3) means 3 counters form blue side and (–3) × (–2) means 3 counters from blue side must be added 2 times but from the opposite side, i.e., from red side.



Therefore, (–3) × (–2) = +6.

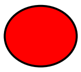
ACTIVITY 9

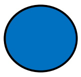
Objective: To divideintegers using counters of different colours.

Materials required: Square or circular counters coloured differently on both the faces/sides, say, one face red and the other face blue.

Procedure:

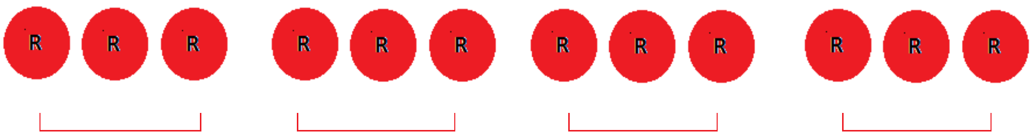
1. Denote the red side of the counter as positive (+) and the blue side of the counter as negative (–)

   i.e.,  or  represents +1

   and  or  represents –1

2. Division of positive integer by a positive integer e.g., 12÷3
   a) To divide 12 by 3, you will subtract 3 from 15 repeatedly to get zero,
      i.e., 12 – 3 = 9, 9 – 3 = 6, 3 – 3 = 0.
      Now, count the number of times you subtract to get zero. It is 4.
      Therefore, 12 ÷ 3 = 4.

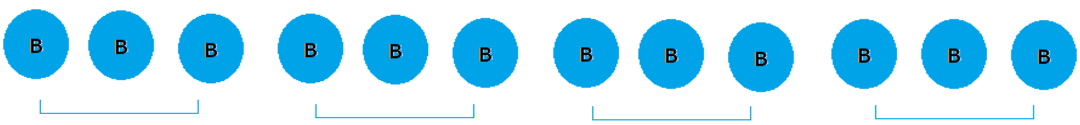
   b) The above can be shown by an activity using counters/cards as follows:
      Place 12 counters from red side in a row. Then pick 3 counters, again pick 3 counters and so until no counter is left. You can easily count that you picked 3 counters from red side 4 times.



1 time 2 time 3 time 4 time

Therefore, 12 ÷ 3 = 4.

3. Division of a negative integer by a positive integer,e.g., (–12) ÷3.
   a) Here, –12 means 12 counters from blue side.Place the 12 blue counters in a row. Then pick 3 counters, again pick 3 counters and so on until no counter is left. You can easily count that you picked 3 counters from blue side 4 times.

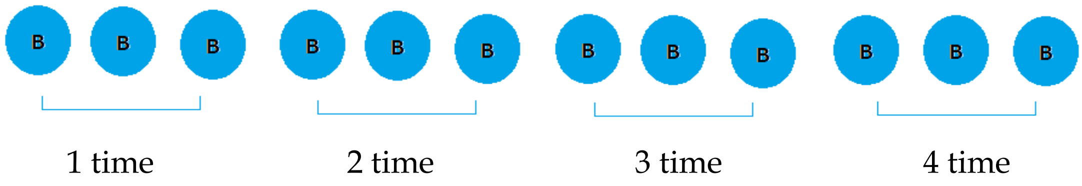


1 time 2time 3 time 4 time

b) Since you pick blue counters 4 times the answer is –4.

Therefore, (–12) ÷ 3 = –4.

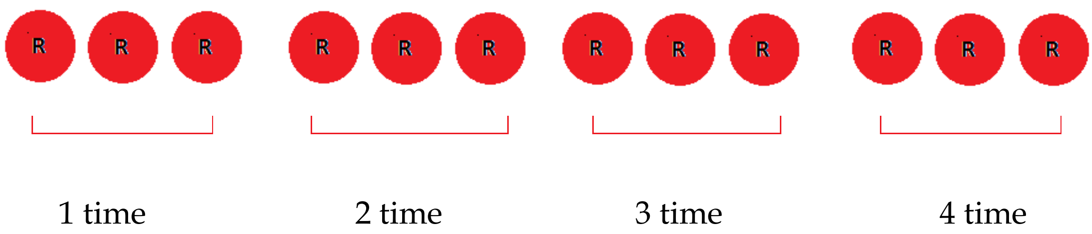
4. Division of positive integer by a negative integer, e.g., 12 ÷ (–3).
   a) Here, 12 means 12 counters from red side, but (–3) means you have to subtract (or pick) 3 counters from the blue side after reversing the red side.
   b) Therefore, you have to place these 12 counters from opposite side,i.e., blue side, and then pick the set of 3 counters repeatedly until no counter is left.



   c) You can easily count that you picked 3 counters from blue side 4 times

   Therefore, 12 ÷ (–3) = –4.

5. Division of negative integer by a negative integer e.g., (–12) ÷ (–3).
   a) Here, (–12) means 12 counters from blue side and (–3) means you have to subtract (or pick) 3 counters from the red side after reversing the blue side.
   b) Therefore, you have to place 12 counters from blue side in a row. Reverse them to get red side and then pick 3 counters from red side repeatedly until no counter is left.



   c) You can easily count that you picked 3 counters from the red side 4 times.

   Therefore, (–12) ÷ (–3) = +4.

ACTIVITY 10

Objective: To add two polynomials. [e.g., $(2x^2 + 3x + 4x) + (x^2 + 4x + 3)$]

Materials required: Cardboard or chart paper, ruler, scissors or cutter, blue and red colours.

Method of construction:

1. Make sufficient numbers of cut-outs (strips) of dimensions x cm × x cm, x cm × 1cm, and 1 cm × 1 cm, as shown in Fig.1.

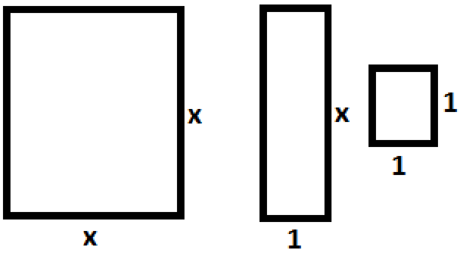


Fig.1

2. Colour one side of each strip by blue colour and the other side by red colour.
3. Let the blue side of the square cut-out of size x cm × x cm represent $+x^2$, the rectangular cut-out of size x cm × 1 cm represent +x, and the square cut-out of size 1 cm × 1 cm represent +1.
4. Similarly, let the corresponding red sides of the cut-outs represent $-x^2$, $-x$ and -1, respectively.

Demonstration:

1. To represent the polynomial $(2x^2 + 3x + 4)$, arrange the strips as shown in Fig.2.

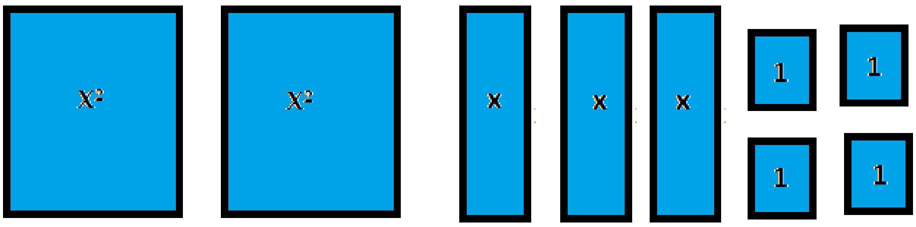


Fig.2

2. Similarly, the polynomial $(x^2 + 4x + 3)$ is represented in Fig.3.

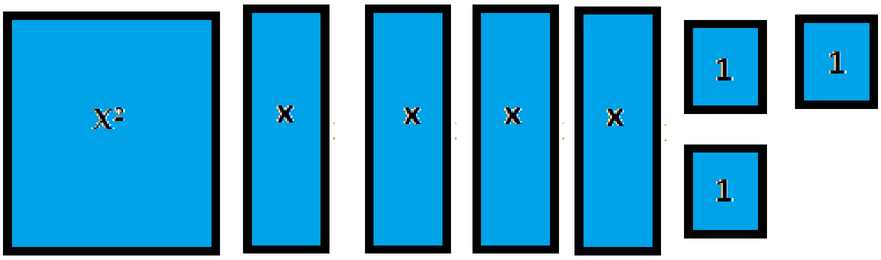


Fig.3

3. To add the above polynomials, combine the strips in Fig.2 and Fig.3, as shown below in Fig.4.

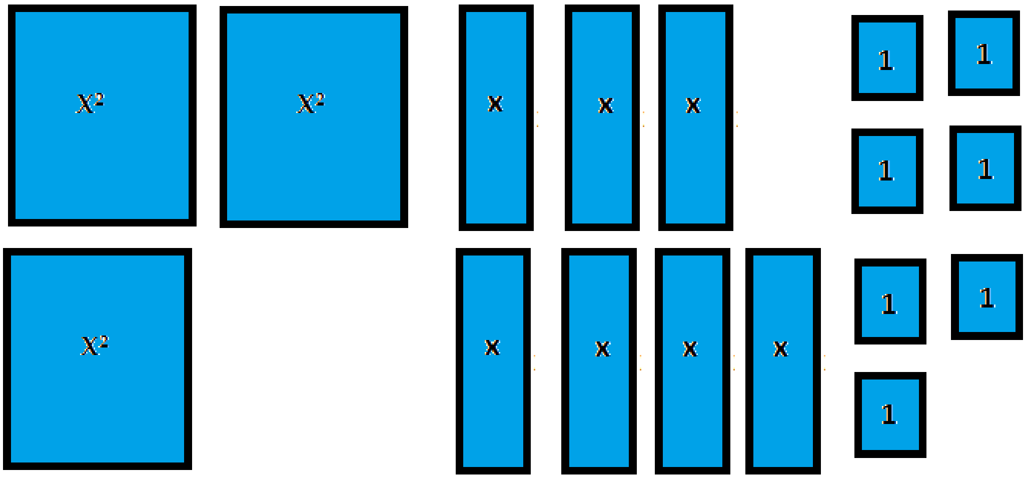


Fig.4

4. Count the strips in Fig. 4, we find that it consists of 3 blue strips of size x cm × x cm, 7 blue strips of size x cm × 1 cm, and 7 blue strips of size 1 cm × 1 cm. (Cancel the strips of the same size in case of different colours).
5. Thus Fig. 4 represents the sum of two polynomials as $3x^2 + 7x + 7$.
6. So, $(2x^2 + 3x + 4) + (x^2 + 4x + 3) = 3x^2 + 7x + 7$.

ACTIVITY 11

Objective: To subtract two polynomials. [e.g., $(2x^2 + 5x - 5x) - (x^2 - 3x - 5)$]

Materials required: Cardboard or chart paper, rulers, scissors or cutters blue and red colour.

Method of construction:

1. Make sufficient numbers of cut-outs (strips) of dimensions x cm × x cm, x cm × 1cm and 1 cm × 1 cm, as shown in Fig.1.

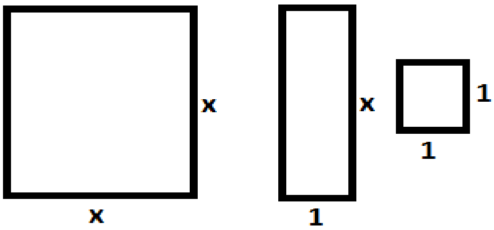


Fig.1

2. Colour one side of each strip by blue colour, and the other side by red colour.
3. Let the blue side of the square cut-out of size x cm × x cm represent $+x^2$, the rectangular cut-out of size x cm × 1 cm represent $+x$, and the square cut-out of size 1 cm × 1 cm represent $+1$.
4. Similarly, let the corresponding red sides of the cut-outs represent $-x^2$, $-x$ and -1, respectively.

Demonstration:

1. To represent the polynomials $(2x^2 + 5x - 2)$, arrange the strips as shown in Fig.2.

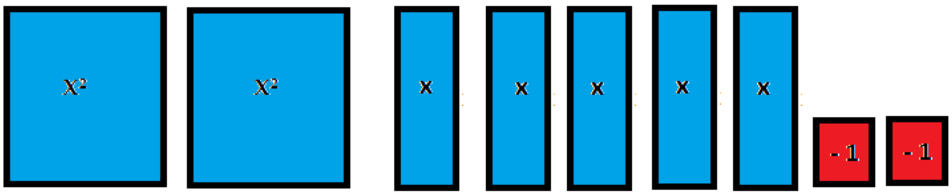


Fig.2

2. Similarly, the polynomial $(x^2 - 3x - 5)$ is represented in Fig.3.

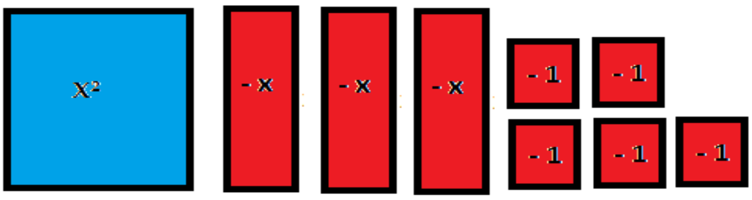


Fig.3

3. To subtract the polynomial ($x^2 - 3x - 5$) from ($2x^2 + 5x - 2$), we have to invert/ reverse each cut-out of Fig.3 and place them along the cut-outs of Fig.2, as shown in Fig.4.

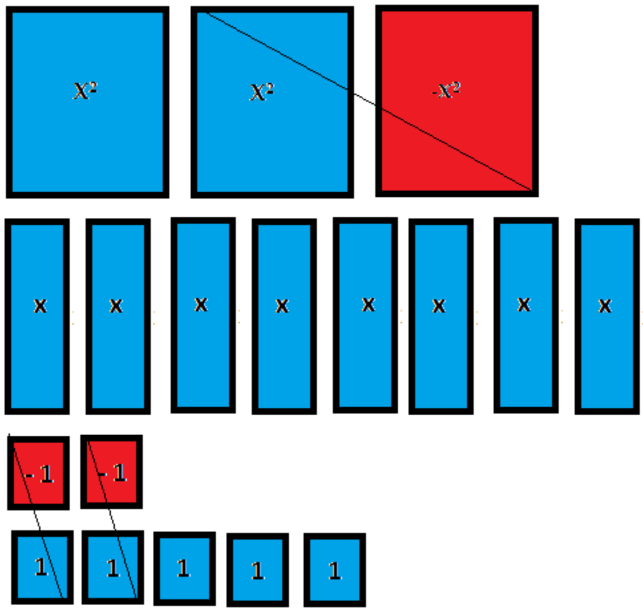


Fig.4

4. Cancel one blue with one red cut-out of the same size, if any, as shown in Fig. 4.
5. The cancelled cut-outs are removed, and the remaining cut-outs represent the polynomial $x^2 + 8x + 3$.
6. Thus, $(2x^2 + 5x - 2) - (x^2 - 3x - 5) = x^2 + 8x + 3$.

ACTIVITY 12

Objective: To factorize polynomials of the type ($ax^2 + bx + c$).

Materials required: Blue and red chart papers/rubber sheets, scissors or cutter.

Method of construction:

1. Make sufficient numbers of blue and red cut-outs (strips) of dimensions x cm × xcm, x cm × 1 cm and 1 cm × 1 cm.
2. Consider each blue square cut-out of size x cm × x cm as $+x^2$, the blue rectangular cut-out of size x cm × 1 cm as +x, and the blue square of cut-out of size 1cm × 1cm as +1.
3. Similarly, let the corresponding red cut outs represent $-x^2$, $-x$ and -1 respectively.

Demonstration:

a) To factorize $2x^2 - 6x$:
   1. Take two blue (x^2) cut-outs and six red ($-x$) cut outs to represent the polynomial $2x^2 - 6$, as shown in Fig.1.

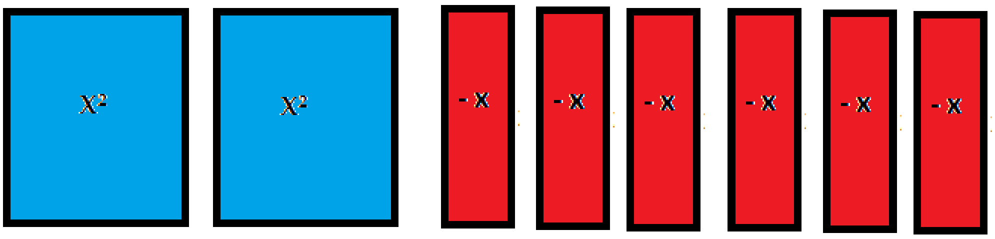


Fig.1

   2. Try to form different types of rectangles/squares using all cut-outs of Fig.1 at a time.

   3. Two different rectangles can be formed as shown in Fig.2 and Fig.3.

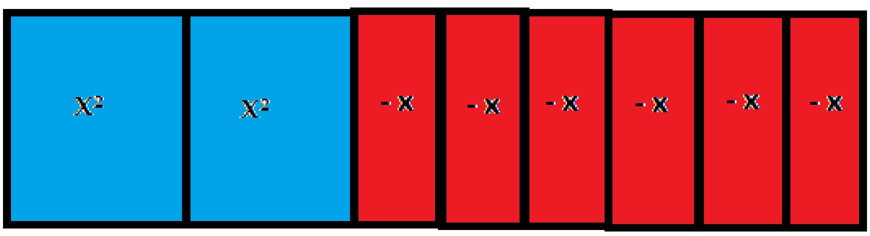


Fig.2

4. The rectangle obtained in Fig.2 has sides of length x and 2x – 6. So, area of this rectangle is x(2x – 6).
   Thus, $2x^2 - 6x = x(2x - 6)$.

5. The rectangle obtained in Fig.3 has sides of length 2x and x – 3. So, area of this rectangle is 2x(x – 3).
   Thus, $2x^2 - 6x = 2x(x - 3)$.

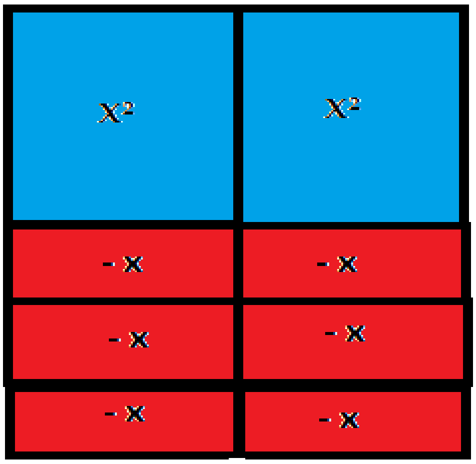


Fig.3

b) To factorize $x^2 + 4x + 4$:

1. Take one blue (x^2) cut-out, four blue (x) cut-outs and four blue unit cut-outs to represent the polynomial $x^2 + 4x + 4$, as shown in Fig.4.

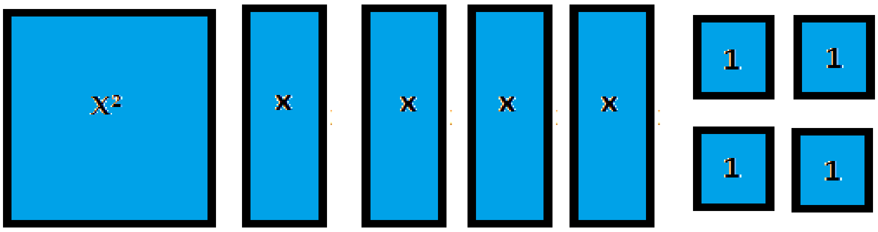


Fig.4

2. Try to form a rectangle or square using all cut-outs at a time as shown in Fig.5.
3. The square obtained in Fig.5 has sides of length (x + 2).
   So, area of this square = $(x + 2)^2$.
4. Also, area of the above square
   = Sum of the areas of all the cut-outs enclosed by the square
   $= x^2 + x + x + x + x + 1 + 1 + 1 + 1$
   $= x^2 + 4x + 4$
5. This shows that, $x^2 + 4x + 4 = (x + 2)^2$.

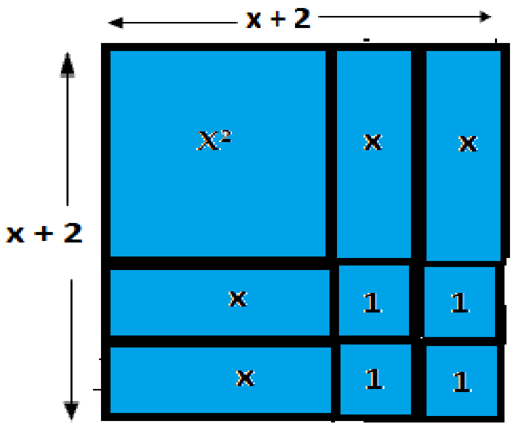


Fig.5

c) To factorize $x^2 - 7x + 12$:

1. Take one blue (x^2) cut-out, seven red ($-x$) cut-outs and twelve unit ($+1$) cut-outs to represent the polynomial $x^2 - 7x + 12$, as shown in Fig.6.

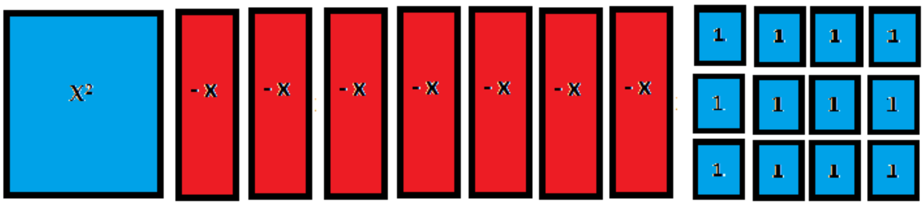


Fig.6

2. Try to form a rectangle or square using all cut-outs at a time as shown in Fig.7.

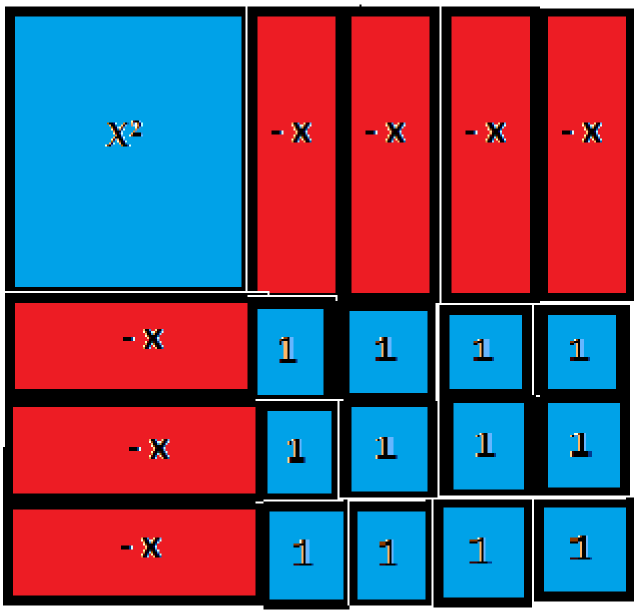


Fig.7

3. The rectangle obtained in Fig.7 has sides of length $(x - 4)$ and $(x - 3)$.
   So, area of this rectangle = $(x - 4)(x - 3)$.

4. Also, area of the above rectangle
   = Sum of the areas of all the cut-outs enclosed by the rectangle
   $= x^2+(-x)+(-x)+(-x)+(-x)+(-x)+(-x)+(-x)+(-x)+1+1+1+1+1+1+1+1+1+1+1+1$
   $= x^2 - 7x + 12$

5. This shows that, $x^2 - 7x + 12 = (x - 4)(x - 3)$.

ACTIVITY 13

Objective: To verify the algebraic identity: $(a + b)^2 = a^2 + 2ab + b^2$.

Materials required: Chart paper or cardboard, or rubber sheets, scissors or cutter, ruler, pencil, sketch pens.

Method of construction:

1. Take a cardboard of convenient size, and cut out a square of side 'a' units and name it as ABCD (Fig.1).
2. Cut out another square of side 'b' units (b<a) and name it as CHGF (Fig.2).

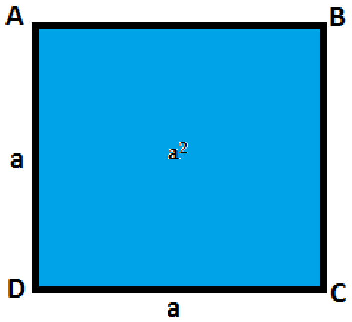


Fig 1

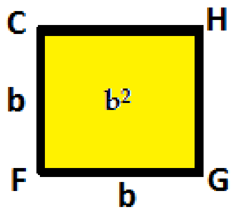


Fig 2

3. Cut out two identical rectangles each of length 'a' units and breadth 'b' units, and name them as DCFE (Fig.3) and BIHC (Fig.4) respectively.

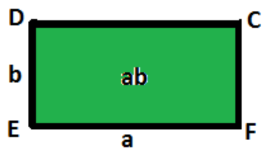


Fig 3

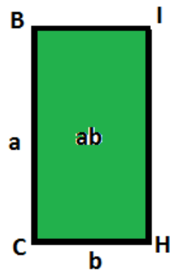


Fig 4

Demonstration:

1. Arrange the four cut-outs on a table as shown in Fig.5. The figure formed is a square AIGE of side (a + b) units.

2. The area of the above square is $(a + b)^2$.

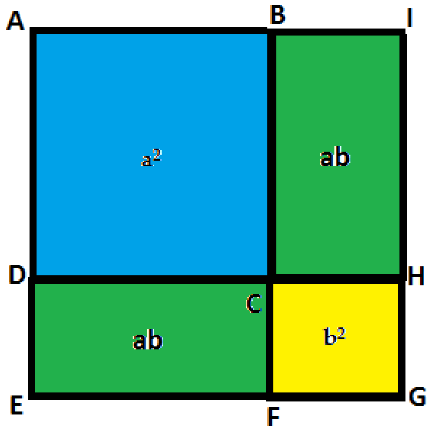


Fig 5

3. Also, the total area of the four cut-outs

   = area of square ABCD + area of rectangle DCFE

   + area of rectangle BIHC + area of square CHGF

   = $a^2 + ab + ab + b^2$

   = $a^2 + 2ab + b^2$

4. Hence, the algebraic identity: $(a + b)^2 = a^2 + 2ab + b^2$ is verified.

ACTIVITY 14

Objective: To verify the algebraic identity $(a - b)^2 = a^2 - 2ab + b^2$.

Materials required: Cardboard or chart paper or rubber sheets, scissors or cutter, ruler, pencil, sketch pens.

Method of construction:

1. Take a cardboard of convenient size, and cut out a square of side 'a' units and name it as ABCD (Fig.1).
2. Cut out another square of side 'b' units (b<a) and name it as DEFG (Fig.2).
3. Cut out two identical rectangles each of length 'a' units and breadth 'b' units, and name them as HIFG (Fig.3) and ABJH (Fig.4) respectively.

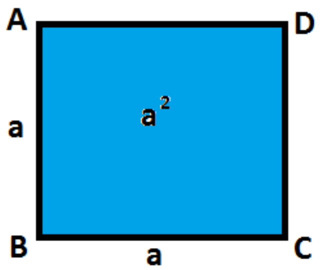


Fig. 1

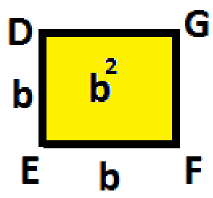


Fig. 2

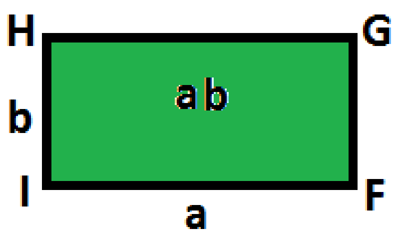


Fig. 3

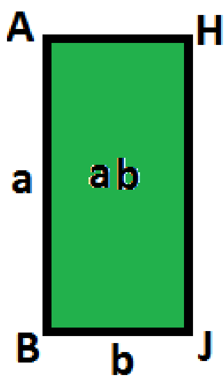


Fig. 4

Demonstration:

1. Arrange the four cut-outs of two squares ABCD and DEFG on a table as shown in Fig.5.

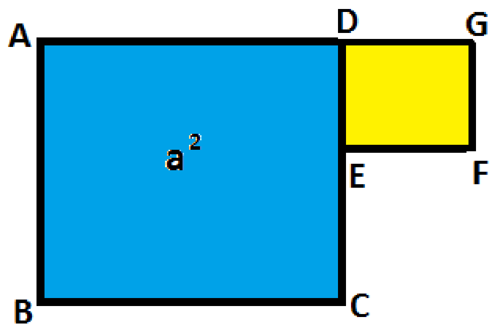


Fig. 5

2. Now put the cut-out of rectangle HIFG on the shape which is obtained in Fig.5 as shown in Fig.6.
3. Put the other cut-out of rectangle ABJH on the shape in Fig.6 as shown in Fig.7.

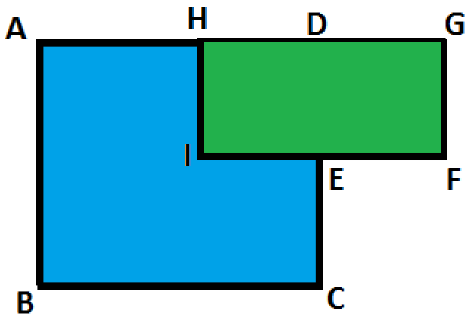


Fig. 6

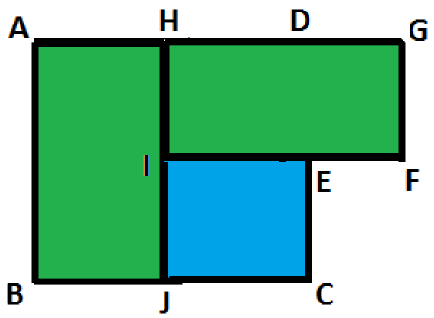


Fig. 7

4. The uncovered portion IJCE in Fig.7 is a square of side (a – b) units. Its area is $(a - b)^2$ sq. units.

5. Now,
   a) the area of the shape given in Fig.5 is $(a^2 + b^2)$ sq. units.
   b) the area of the shaded/covered portion given in Fig.7 is 2ab sq. units.
   c) the area of the uncovered portion given in Fig.7 from steps 1, 2 and 3 is $(a^2 + b^2 - 2ab)^2$ sq. units.

6. Hence, the algebraic identity: $(a - b)^2 = (a^2 - b^2 + 2ab)^2$ is verified.

ACTIVITY 15

Objective: To verify the algebraic identity: $a^2 - b^2 = (a + b)(a - b)$.

Materials required: Cardboard or chart paper, or rubber sheets, scissors or cutter, ruler, pencil, sketch pens.

Method of construction:

1. Take out a cardboard of convenient size, and cut out a square of side 'a' units and name it as ABCD.
2. Cut out a square AEFG of side 'b' units ($b<a$) from the square ABCD.We are left with a figure EBCDGFE(Fig.1).
3. Join FC. Cut out FC to get two congruent trapeziums EBCF and GDCF (Fig.1).

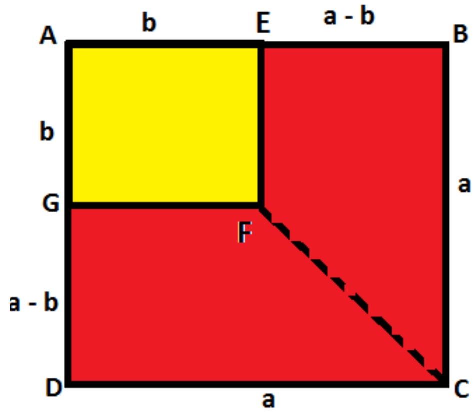


Fig 1

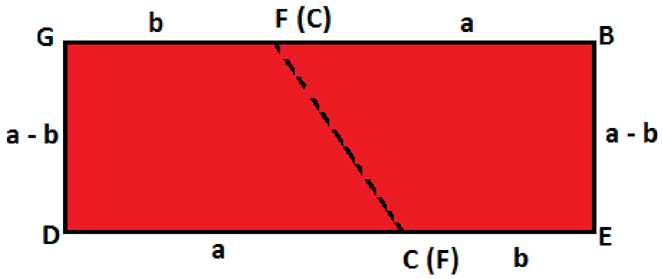


Fig 2

Demonstration:

1. First, arrange the square AEFG of side 'b' units, the two trapeziums EBCF and GDCF to get a square ABCD of side 'a' units (as shown in Fig.1).
2. Remove the square AEFG. We are left with the two trapeziums EBCF and GDCF.
3. Now, arrange the two cut-outs of trapeziums to form a rectangle GBED of length $(a + b)$ units and breadth $(a - b)$ units, as shown in Fig.2.
4. InFig.1,

   Area of square ABCD – Area of square AEFG

   =Area of trapezium EBCF + Area of trapezium GDCF

   =Area of rectangle GBED (Fig.2)

   = GB × GD

Thus, $a^2 - b^2 = (a + b)(a - b)$

ACTIVITY 16

Objective: To obtain the angle bisector of an angle by paper folding.

Materials required: Chart paper or any sheet of paper, pencil, pen, ruler, scissors.

Method of construction:

1. Take a chart paper or a sheet of paper of convenient size, and mark a point O on it.
2. With O as initial point, draw two rays $\overrightarrow{OA}$ and, $\overrightarrow{OB}$making $\angle AOB$.
3. Cut out the $\angle AOB$ from the chart paper/sheet of paper.
4. Fold the $\angle AOB$ through the vertex O such that ray $\overrightarrow{OA}$ falls along ray $\overrightarrow{OB}$.

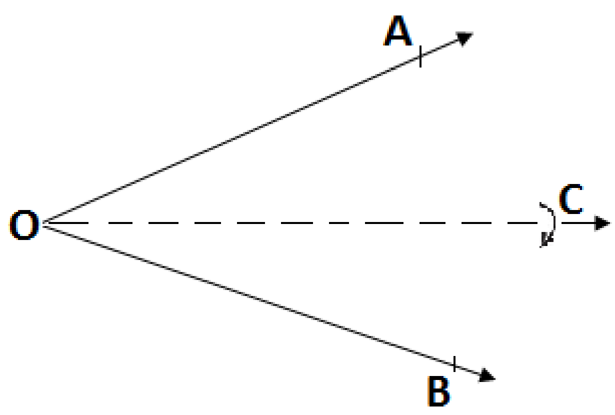


Fig 1

5. Unfold the $\angle AOB$, and then mark a point C on the crease (as shown in Fig.1).
6. Make cut outs of $\angle AOC$ and $\angle COB$.

Demonstration:

1. Place the cut out of $\angle AOC$ on $\angle COB$ (or the cut out of $\angle COB$ on $\angle AOC$).
2. The $\angle AOC$ exactly covers $\angle COB$ (or $\angle COB$ covers exactly $\angle AOC$).

Observation: On the actual measurement,

Measure of $\angle AOC$= ___________

Measure of $\angle COB$= ___________

∴$\angle$AOC and $\angle$OCB are _____________

Conclusion:

Since, $\angle$AOC and $\angle$COB are equal,

Hence, OC is the angle bisector of $\angle$AOB.

ACTIVITY 17

Objective: To verify exterior angle property of a triangle.

Materials required: Chart paper, plain sheet of paper, colours, adhesive, scissors, ruler, pen/pencil.

Method of constructions:

1. Make/Cut out identical triangles from a chart paper.
2. Colour the angles of the two triangles (as shown in Fig.1).

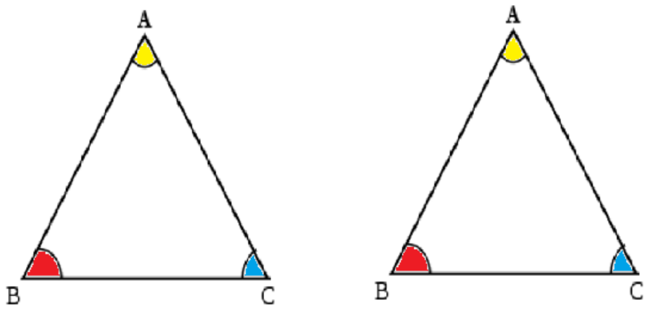


Fig 1

3. Paste one of the triangles on a sheet of paper/chat paper, and produce one of its sides, say BC, to D forming exterior ∠ACD (as shown in Fig.2).

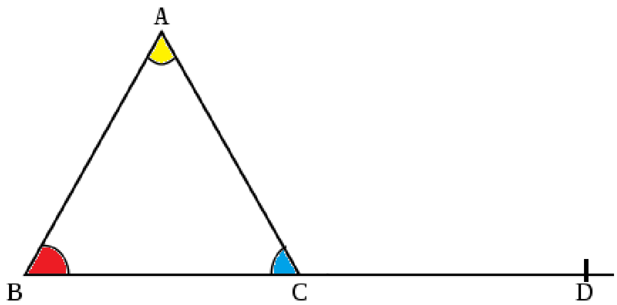


Fig 2

4. Now, cut out ∠A and ∠B from the other triangle.

Demonstration:

1. Place the cut outs of ∠A and ∠B on the exterior ∠ACD such that there is no gap between the two cuts outs (as shown in Fig.3).

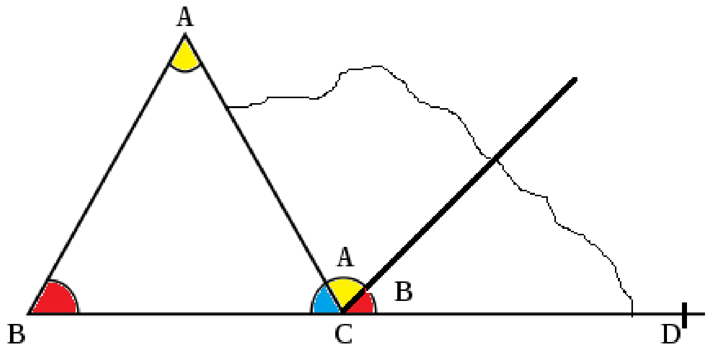


Fig 3

2. The two angles, ∠A and ∠B which are the interior opposite angles of exterior ∠ACD, together exactly cover∠ACD (as shown in Fig.3).

Observation: On actual measurement,

Measureof ∠A =

Measure of ∠B =

Measure of ∠ACD =

∴ ∠A +∠B = + =

= ∠ACD

Conclusion:

Since ∠ACD = ∠A + ∠B

Thus, the exterior angle of a triangle is equal to the sum of its two interior opposite angles.

ACTIVITY 18

Objective: To verify the angle sum property of a triangle (i.e., to verify that the sum of three angles of a triangle is 180°.

Materials required: Chart paper/Card board, plain sheet of paper, colours, scissors, ruler, pen/pencil.

Method of construction:

1. Make/Cut out two identical triangles from a chart paper/cardboard.
2. Colour the angles of the two triangles and name the angles as 1, 2, and 3 (as shown in Fig.1).

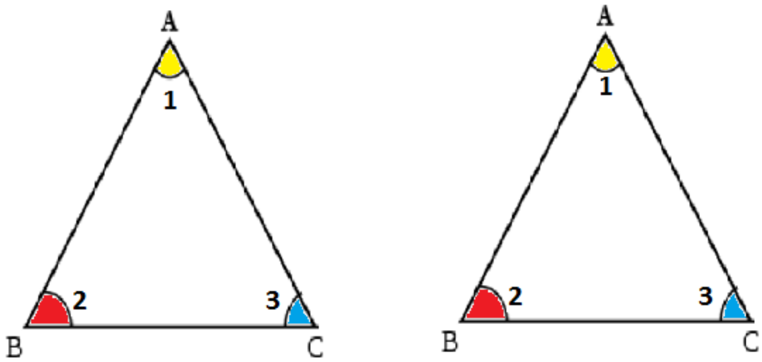


Fig 1

3. Cut out the three angles of one triangle (as shown in Fig. 2).

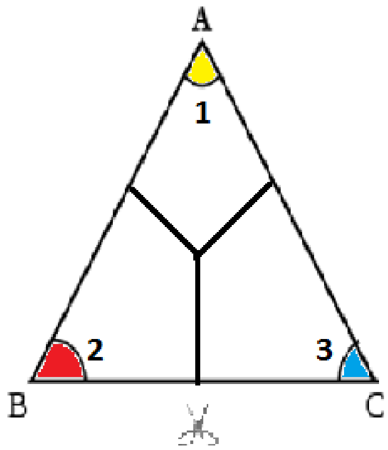


Fig 2

Demonstration:

1. Place the three cut-outs on a sheet of paper adjacent to each other with their vertices A, B and C at a common point O, so that there is no gap between them (as shown in Fig.3).

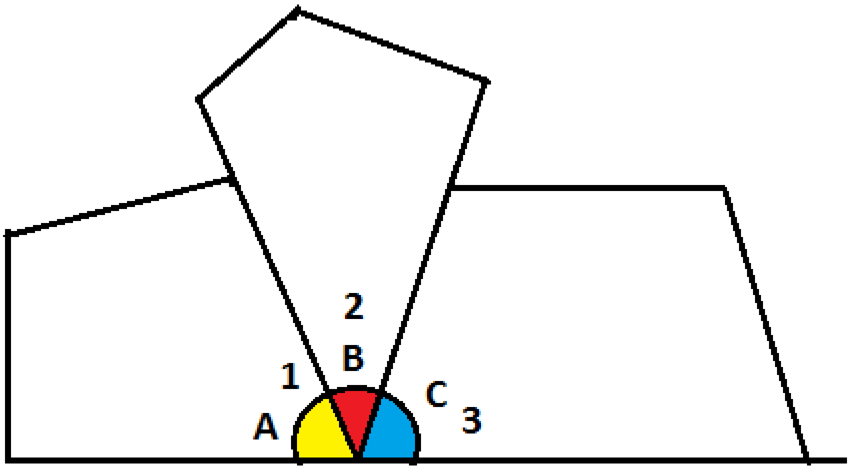


Fig 3

2. The cut-outs of the three angles A, B and C placed adjacent to each other at a common vertex O form a straight angle.

   Therefore, $\angle A + \angle B + \angle C = 180^{o}$

Observation: On actual measurement,

Measure of $\angle A$	=__________
Measure of $\angle B$	=__________
Measure of $\angle C$	=__________
$\therefore \angle A + \angle B + \angle C$	= ______ +______+______
	= __________

Conclusion: Thus, the sum of the three angles of a triangle is 180^{o} or two right angles.

ACTIVITY 19

Objective: To verify that the sum of four angles of a quadrilateral is 360º.

Materials required: Chart paper/Card board, plain sheet of paper, colours, scissors, ruler, pen/pencil.

Method of construction:

1. Make/Cut out two identical quadrilaterals from a chart paper/cardboard.
2. Colour the angles of the two quadrilaterals and name the angles as 1,2, 3 and 4 (as shown in Fig.1).

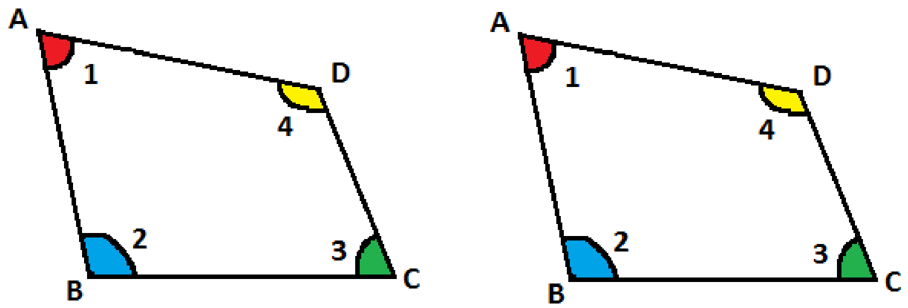


Fig 1

3. Cut out the four angles of one quadrilateral (as shown in Fig.2)

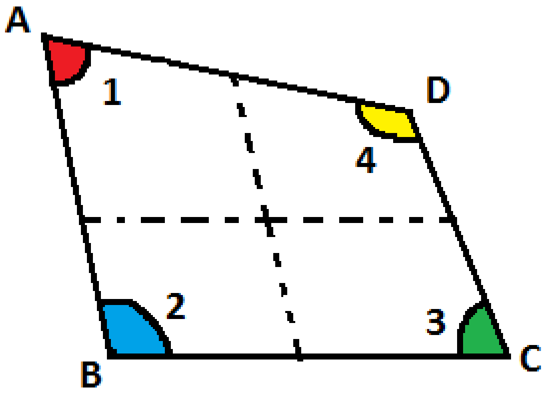


Fig 2

Demonstration:

1. Place the four cut-outs on a sheet of paper adjacent to each other with their vertices A, B, C and D at a common point O, without leaving any gap between them (as shown in Fig.3).

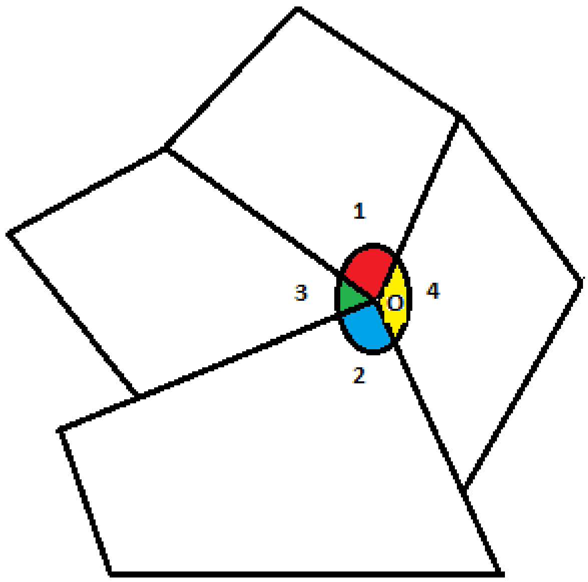


Fig 3

2. The cut outs of the fourangles A, B, C and D make a complete angle at the point O.

   Therefore, $\angle A + \angle B + \angle C + \angle D = 360^o$.

Observation: On actual measurement,

Measure of$\angle A$ =___________

Measure of $\angle B$ =___________

Measure of $\angle C$ =___________

Measure of $\angle D$ =___________

$\therefore$ $\angle A + \angle B + \angle C + \angle D$ = ______ + ______ +_______+______

= ____________

Conclusion: Thus, the sum of the four angles of a quadrilateral is 360^o.

ACTIVITY 20

Objective: To get a median of a triangle from any vertex, and to verify that medians of a triangle meet at a point.

Materials required: Chart paper/sheet of paper, pen/pencil scissors.

Procedure:

1. Cut out a triangular shape from a chart paper/sheet of paper, and name it as ABC (Fig.1).
2. Find the mid-point of side BC by folding the paper such that the vertex C falls on vertex B. Mark the point of intersection of the line of fold with BC as D, which is the mid-point of BC (Fig.2).

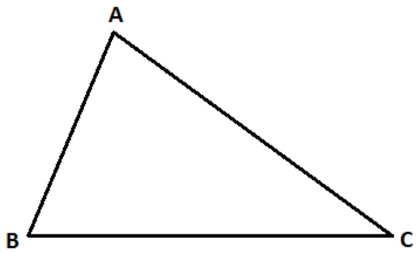


Fig 1

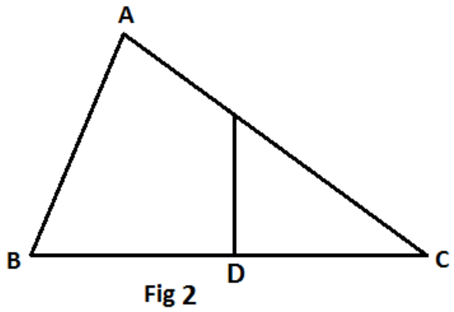


Fig 2

3. Similarly, get the mid-points of CA and AB and mark them as E and F respectively.
4. Now fold the triangle to create a crease along AD. The crease thus obtained is the median of the triangle from vertex A on the side BC.
5. Similarly, get the medians from vertex B and C as BE and CF respectively by paper folding (as shown in Fig.3).

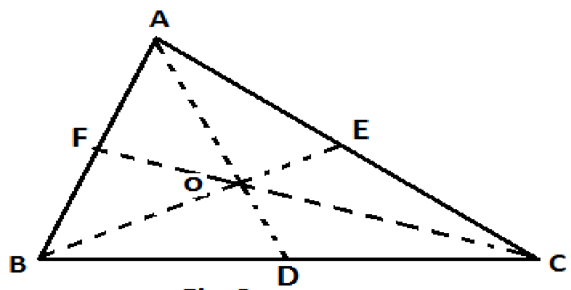


Fig 3

6. The three creases AD, BE and CF may be marked with a pencil (Fig.3).

Observation:

1. The three medians AD, BE and CF meet at a point O.
2. The point O lies in the interior of ΔABC.

Conclusion: The three medians of a triangle meet at a point.

ACTIVITY 21

Objective: To verify the Pythagoras Theorem, i.e., "In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides".

Materials required: Card board/Sheet of paper, scissors/cutter, pen/pencil/sketch pens.

Method of construction:

1. Make/Cut out from a card board, fouridentical right angled triangles each of sides a, b and c and name them at T_1, T_2, T_3 and T_4 (Fig.1).

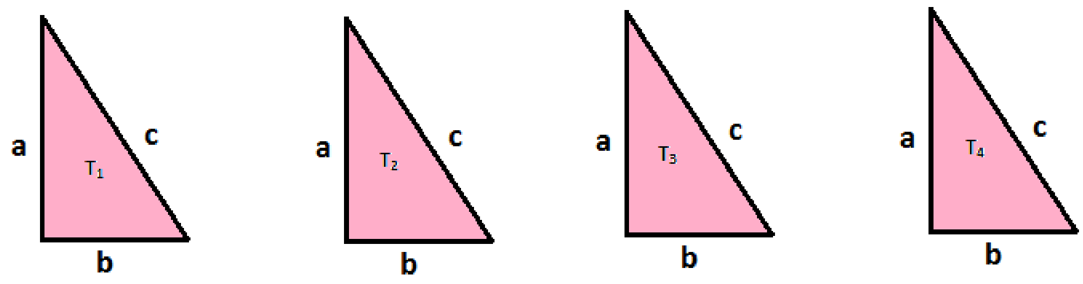


Fig 1

2. Make/Cut out from a card board,three squares (a square of side a, a square of side b and a square of side c).
3. Draw on a sheet of paper, a square outline with side (a+b).

Demonstration:

1. Place the four triangles and a square of side c (i.e., c^2) within the square outline of side (a + b), as shown in Fig.2.
2. Remove the square of side c. Take the two squares of sides a and side b (i.e., a^2 and b^2) and arrange them together with the four triangles within the same square outline of side (a + b), as shown in Fig.3.

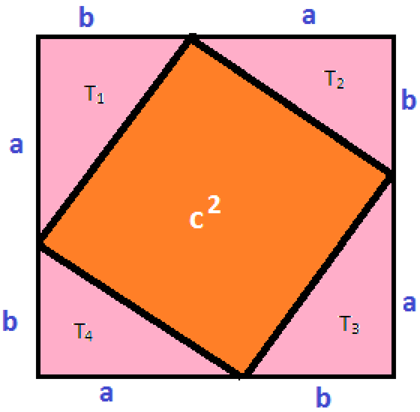


Fig 2

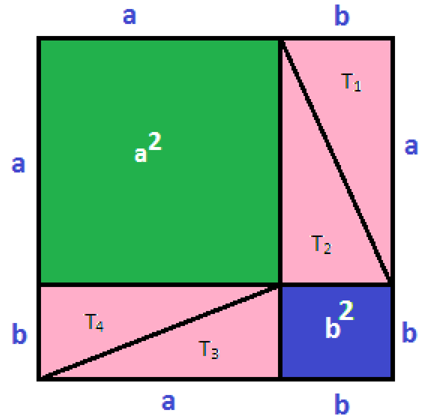


Fig 3

Observation:

1. In Fig.2, the square outline is covered by four identical triangles (T_1, T_2, T_3 and T_4) and a square of side 'c'.
2. In Fig.3, the same square outline is covered by the same four identical triangles (T_1, T_2, T_3 and T_4), a square of side 'a' and a square of side 'b'.
3. The area covered by a square of side 'c' (i.e., c^2) is the same as the area covered by a square of side 'a' (i.e., a^2) and the area covered by a square of side 'b' (i.e., b^2).
4. Thus shows that, $c^2 = a^2 + b^2$.

Conclusion: Thus, "In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides".

This verifies the Pythagoras Theorem.

ACTIVITY 22

Objective: To verify Pythagoras Theorem (using squared papers)

Materials required: Squared papers, sheet of paper, pen/pencil/sketch pens.

Procedure:

1. Draw a right angled triangle ABC, right angled at B, of sides, say, AB= 4 units, BC = 3 units and CA= 5 units on a sheet of paper (Fig.1).
2. Make three squares of sides 3 units, 4 units and 5 units on a squared paper of the same type and cut them out.
3. Pastethe cut out square of side 3 units along the side BC, the cut out square of side 4 units along the side AB and the cut out square of side 5 units along the side CA ofthe triangle (as shown in Fig.2).

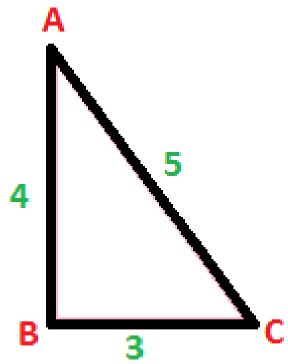


Fig 1

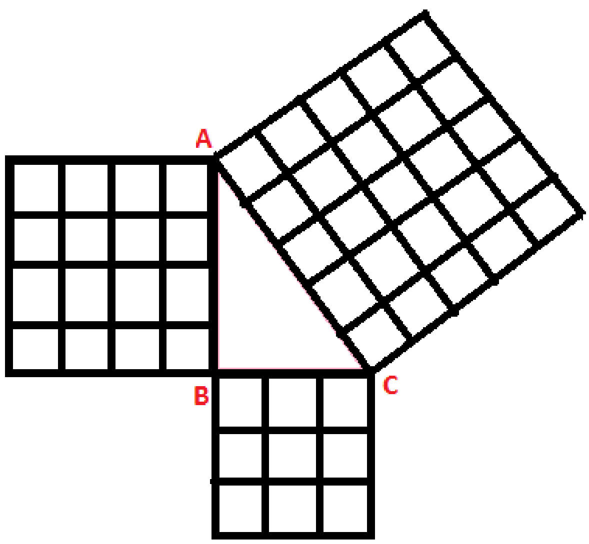


Fig 2

4. Count the number of unit squares in each of the three squares on AB, BC and CA.

Observation:

1. Number of unit squares in the square on side AB = ________.
2. Number of unit squares in the square on side BC = ________.
3. Number of unit squares in the square on side CA = ________.

4. Sum of number of unit squares in the squares on sides AB and BC

= _______ + _______

= ________.

5. Number of unit squares on side _______

= sum of number of unit squares on sides ________ and _________.

Conclusion: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

ACTIVITY 23

Objective: To find the ratio of the circumference and diameter of a circle.

Materials required: Thick paper/Cardboard, sheet of paper, pen, pencil, eraser, compass, scissors.

Procedure:

1. Draw a circle on a cardboard/thick paper and cut it out.
2. Fold it into two equal parts to obtain a crease.
3. Join with a pencil along the line of folding to get a line segment AB (Fig. 1).

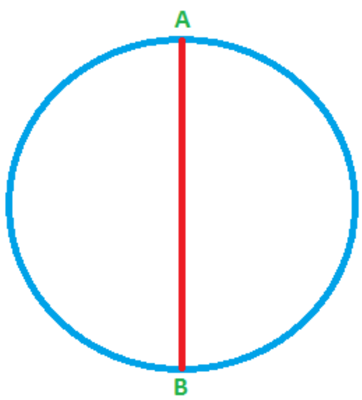


Fig 1

4. Draw a ray in a sheet of paper and mark its initial point as P.
5. Hold the circular disc such that the point A on the circle coincides with the point P on the ray (Fig.2).

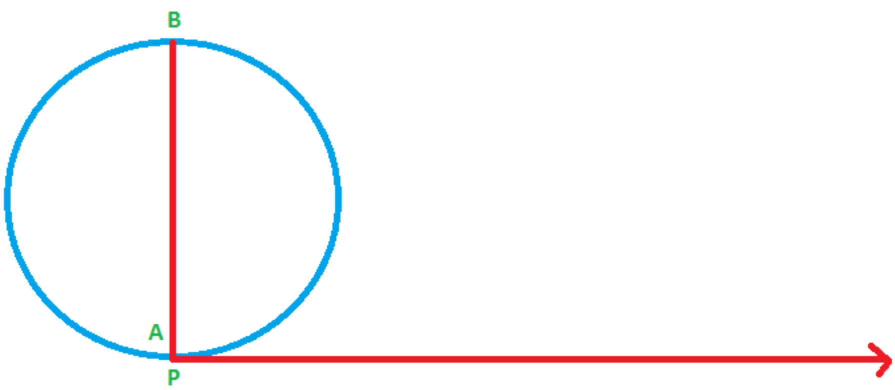


Fig 2

6. Rotate the circular disc along the ray till the point A again touches the ray. Mark that point on the ray as Q (Fig.3).

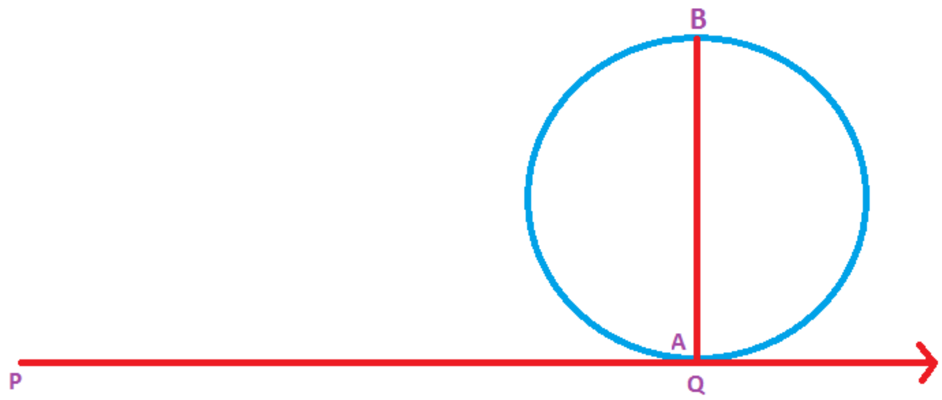


Fig 3

7. Measure AB and PQ.
8. Repeat the above process for circles of different radii.

Observation:

1. The line segment AB is the diameter (d) of the circle.
2. The length PQ gives the circumference (c) of the circle.
3. On measurement,

   AB = d = ________

   PQ = c = ________
4. Find the ratio $\frac{c}{d}$ for circles of different radii, and complete the following table:

Circle	Diameter (d)	Circumference (c)	Ratio $\frac{c}{d}$
1			
2			
3			

Conclusion:

1. Each time, the ratio $\frac{c}{d}$ is constant. This constant is denoted by the symbol π.
2. Value of $\pi = \frac{c}{d} =$ ______________ approximately.

ACTIVITY 24

Objective: To obtain/explore the formula for the area of a rectangle.

Materials required: Squared paper/dot paper, pen/pencil, ruler.

Procedure:

1. Form shapes of different rectangles on a squared paper/dot paper (as shown in Fig.1).
2. Count the number of unit squares enclosed in each rectangle.
3. Find/Measure the length and breadth of each rectangle.

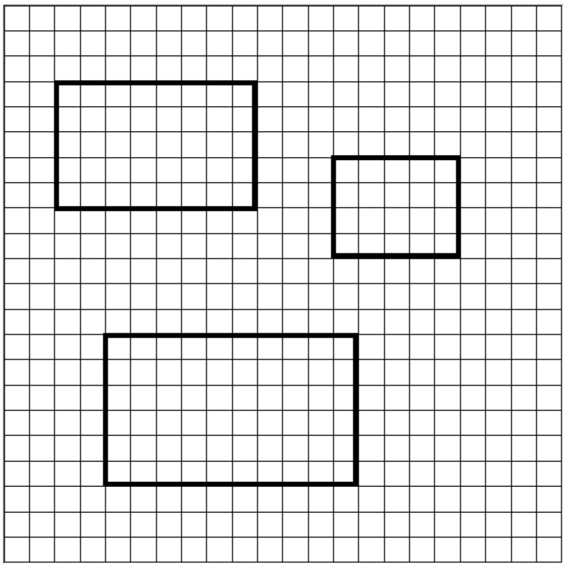


Fig 1

4. Complete the following table:

Sl.No.	Total number of unit squares in rectangle	Length of the rectangle	Breadth of the rectangle	Length × Breadth
1	20	5	4	5 × 4 = 20
2				
3				

Observation:

1. The total number of unit squares in the rectangle gives the area of the rectangle.
2. Every time, the product of length and breadth of the rectangle, i.e., length × breadth, is the same as the area of the rectangle by counting the unit squares.

Conclusion: The area of rectangle = ________________ × ________________

ACTIVITY 25

Objective: To obtain/explore the formula for the area of a parallelogram.

Materials required: Cardboard/thick paper, cutter/scissors, pen/pencil.

Method of construction:

1. Draw a parallelogram on a cardboard and name it as ABCD (Fig.1).
2. Through A, draw AE ⊥DC. Mark the ΔADE as P and the trapezium ABCE as Q (as shown in Fig.1).

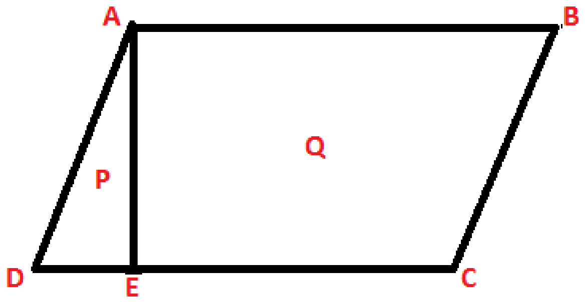


Fig 1

3. Cut out the parallelogram from the cardboard and the triangle P from the parallelogram.

Demonstration:

1. Remove the cut-out P of triangular piece.
2. Place/Attach P to the other side of the cut-out Q of the trapezium piece (as shown in Fig.2).

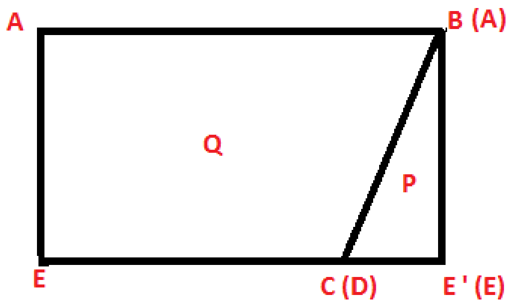


Fig. 2

3. The figure ABE'Eformed (in Fig.2) is a rectangle.

Observation:

1. The length of the rectangle ABE'E is equal to the base of the parallelogram ABCD.
2. The breadth of the rectangle ABE'E is equal to the height of the parallelogram ABCD.
3. Area of the parallelogram ABCD = Area of the rectangle ABE'E
   = Length × Breadth of rectangle ABE'E
   = Base × Height of parallelogram ABCD

Conclusion: Area of parallelogram = base × height.

ACTIVITY 26

Objective: To obtain/explore the formula for the area of a triangle.

Materials required: Cardboard/thick paper, cutter/scissors, pen/sketch pens.

Procedure:

1. Make/Cut out from a cardboard two congruent triangles and mark them as T_1 and T_2.
2. Place together the cut outs of the two congruent triangles T_1 and T_2 to form a parallelogram (as shown in Fig.1).

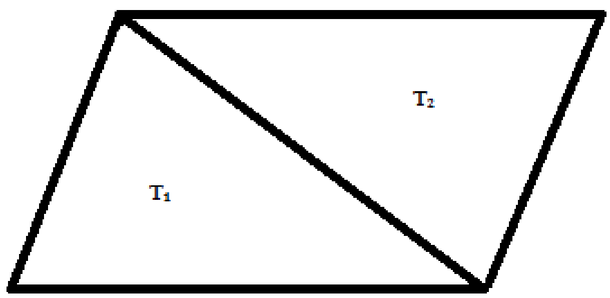


Fig 1

Observation:

1. Since T_1 and T_2 are identical, the area of T_1 or $T_2$is half the area of the whole figure, which is a parallelogram.
2. Thus, area of T_1 or T_2 $= \frac{1}{2} \times$ Area of the parallelogram.

   $= \frac{1}{2} \times$ base $\times$ height.

Conclusion: Area of triangle $= \frac{1}{2} \times$ base $\times$ height.

ACTIVITY 27

Objective: To find the formula for the area of a trapezium.

Materials required: Cardboard, cutter/scissors, pen/pencil, sketch pens.

Procedure:

1. Make cut-outs of two congruent trapeziums each with parallel sides 'a' and 'b' and height 'h' (as shown in Fig.1).

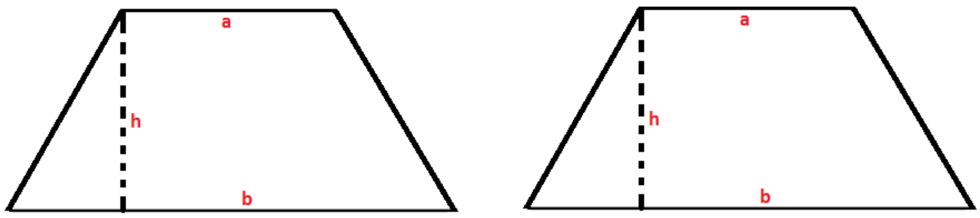


Fig 1

2. Place the two-cuts together to form a parallelogram ABCD (as shown in Fig.2).

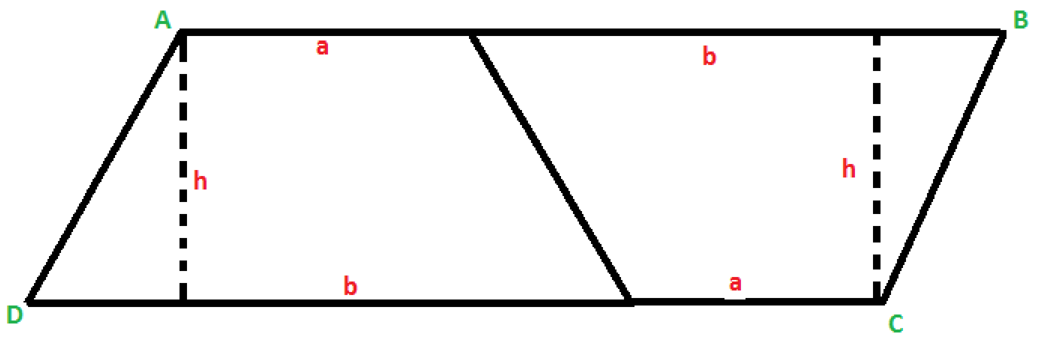


Fig 2

Observation:

1. Figure formed by the two identical trapeziums is a parallelogram.
2. The base of the parallelogram = (a+b).
3. The height of the parallelogram = height of each of the trapezium

   = h

4. Since the two trapeziums are identical, the area of each trapezium is half the area of the whole figure, which is a parallelogram.

5. Thus, area of each trapezium $= \frac{1}{2} \times$ (area of the parallelogram)

$= \frac{1}{2} \times$ (base × height)

$= \frac{1}{2} \times (a + b) \times h$

Conclusion: Area of trapezium $= \frac{1}{2} \times (a + b) \times h$

$= \frac{1}{2}$ (sum of parallel sides) × perpendicular distance betweenparallel sides.

ACTIVITY 28

Objectives: To obtain/explore a formula for the area of a circle.

Materials required: Cardboard, compass, protractor, pencil, pen, ruler.

Methods of construction:

1. Take a cardboard and on it draw four circles each of radius 'a' (say 8 cm).
2. Make cut-outs of these four circles.
3. Divide the first circular cut-out into 6 cut-outs of equal sectors and label half of them, i.e., 3 sectors, as 'A'.
4. Divide the second circular cut-out into 8 cut-outs of equal sectors and label half of them, i.e., 4 sectors, as 'B'.
5. Divide the third circular cut-out into 12 cut-outs of equal sectors and label half of them, i.e., 6 sectors, as 'C'.
6. Divide the fourth circular cut-out into 16 cut-outs of equal sectors and label half of them i.e., 8 sectors, as 'D'.

Demonstration:

1. Take the 6 equal sectors, 3 of which are labelled as A, and arrange them to form a circle, as shown in Fig.1.
2. Now, re-arrange the sectorsto form a shape as shown in Fig.2.

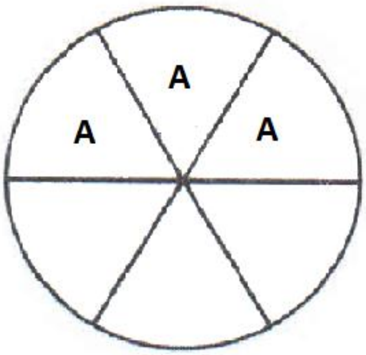


Fig. 1

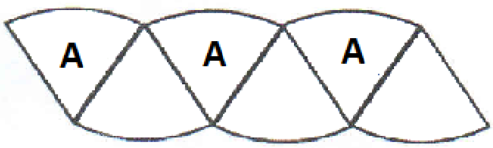


Fig. 2

3. Take the 8 equal sectors, 4 of which are labelled as B, and arrange them to form a circle as shown in Fig.3.

4. Rearrange the sectors to form a shape as shown in Fig.4.

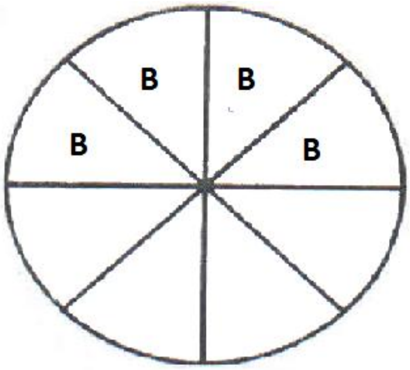


Fig. 3

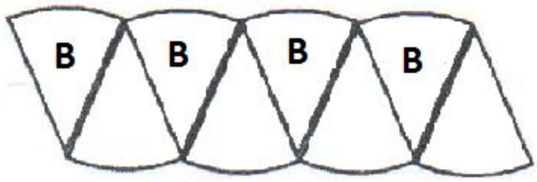


Fig. 4

5. Take the 12 equal sectors, 6 of which are labelled as C, and arrange them to form a circle as shown in Fig.5.
6. Rearrange the sectors to form a shape as shown in Fig.6.

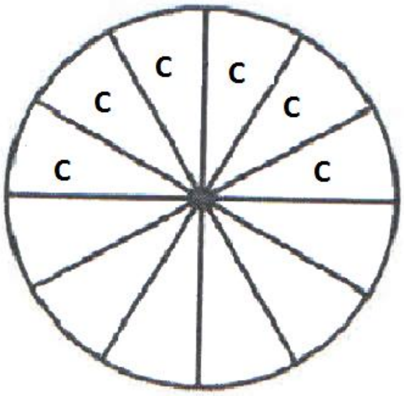


Fig. 5

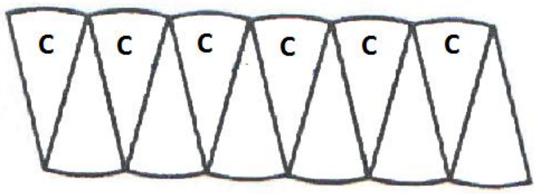


Fig. 6

7. Take the 16 equal sectors, 8 of which are labelled as D, and arrange them to form a circle as shown in Fig.7.
8. Rearrange the sectors to form a shape as shown in Fig.8.

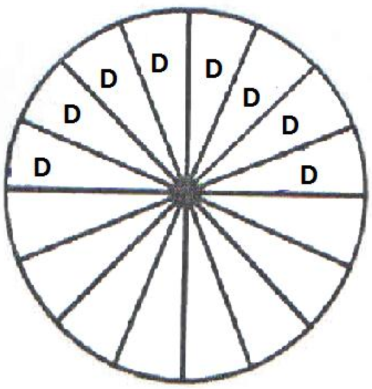


Fig. 7

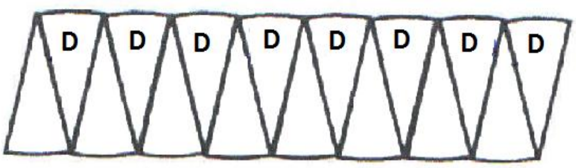


Fig. 8

Observation:

1. Observe figures 2, 4, 5 and 8.
2. As number of equal sectors of the circle is increasing, the shape of the figure is becoming /nearly a rectangle.
3. The length of the rectangle $= \frac{1}{2}$ of circumference of the circle
   $= \frac{1}{2} \times (2\pi r)$
   $= \pi r$
4. The breadth of the rectangle = radius of the circle
   = r
5. Thus, area of the circle = Area of the rectangle
   = length x breadth
   $= \pi r \times r$
   $= \pi r^2$

Conclusion: Area of a circle with radius 'r' $= \pi r^2$.

ACTIVITY 29

Objectives: To form a cuboid and obtain a formula for its surface area.

Materials required: Chart paper, scissors, ruler, pencil, sketch pen, cello tape.

Procedure:

1. Using a chart paper, make a shape involving two identical rectangles of dimensions 'l' units × 'b' units, two identical rectangles of dimensions 'b' units × 'h' units, and two identical rectangles of dimensions 'h' units × 'l' units, as shown in Fig.1.

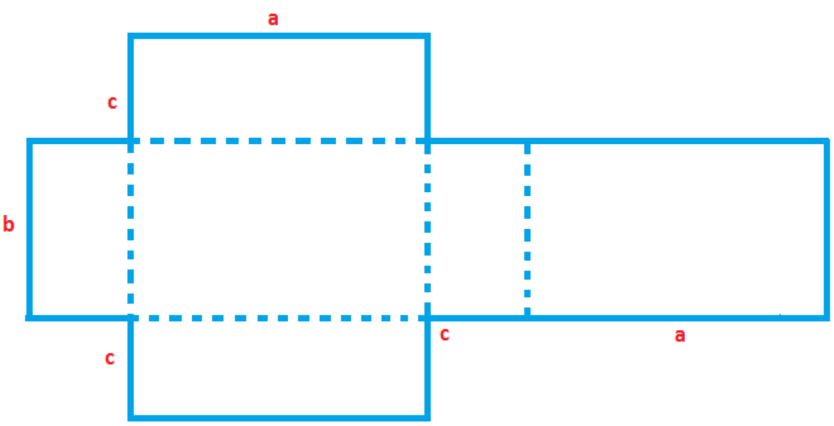


Fig 1

2. Fold the 6 rectangles along the dotted lines to obtain a 3-dimensional shape as shown in Fig.2. (may use cello tape if necessary)

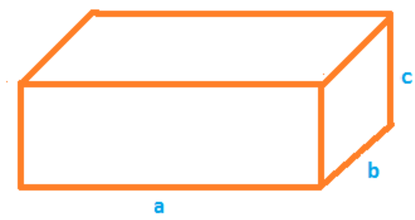


Fig 2

Observation:

1. The 3–dimensional shape obtained in Fig.2is a cuboid.
2. Area of two rectangles each of dimensions l units × b units = 2 × lb
3. Area of two rectangles each of dimensions b units × h units = 2 x bh
4. Area of two rectangles each of dimensions h units × l units = 2 x hl
5. The surface area of the cuboid = 2 ×lb + 2 ×bh+2 × hl
   = 2 (lb + bh+ hl)

Conclusion: Therefore, the surface area of the cuboid = 2 (lb+bh+hl)

*[**Note:** The shape in Fig.1 is called a Net of Cuboid].*

ACTIVITY 30

Objectives: To find a formula for the total surface area of a right circular cylinder.

Materials required: Chart paper/thick paper, scissors, ruler, compass, pencil, cello tape.

Procedure:

1. Using a chart paper, make a shape involving two identical circles, each of radius 'r', and a rectangle of length 'l' and breadth 'h' as shown in Fig.1. The length of the rectangle is in such a way that it is equal to the circumference of each of the two circles.
2. Fold the shape in Fig.1 along its breadth and joins the two ends, the circular base and the circular lid by using cello tape to obtain a 3-dimensional shape as shown in Fig.2.

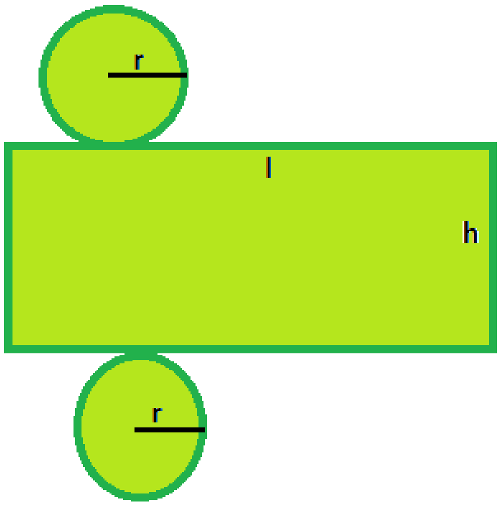


Fig 1

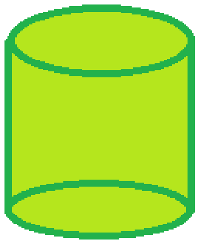


Fig 2

Observation:

1. The 3-dimensional shape obtained in Fig.2 is a right circular cylinder having a base and a lid.
2. Radius of circular base/lid = r = radius of the cylinder.
3. Breadth of rectangle = b = height of the cylinder.
4. Length of rectangle = l = circumference of the base of the cylinder = $2\pi r$
5. Area of circular base/lid = πr^2

6. Area of curved surface area of cylinder = area of rectangle

$= l \times h = 2\pi r \times h$

$= 2\pi rh$

Conclusion: Thus, the total surface area of a cylinder

= area of rectangle + area of 2 circles

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$

*[**Note:** The shape in Fig. 1 is called a Net of Cylinder)*

BEAUTY OF MATHEMATICS

Looking for patterns is very much a part of Mathematics. And through patterns we can see how Mathematics is so beautiful!

Number 1

Look at this symmetry:

1	×	1	=	1
11	×	11	=	121
111	×	111	=	12321
1,111	×	1,111	=	1234321
11,111	×	11,111	=	123454321
1,11,111	×	1,11,111	=	12345654321
11,11,111	×	11,11,111	=	1234567654321
1,11,11,111	×	1,11,11,111	=	123456787654321
11,11,11,111	×	11,11,11,111	=	12345678987654321

1	×	1	=	1
11	×	1	=	11
111	×	11	=	1221
1,111	×	111	=	123321
11,111	×	1,111	=	12344321
1,11,111	×	11,111	=	1234554321
11,11,111	×	1,11,111	=	123456654321
1,11,11,111	×	11,11,111	=	12345677654321
11,11,11,111	×	1,11,11,111	=	1234567887654321

Number 5

Look at the squares of numbers ending in 5:

5×5 or $(5)^2$	=	25 (0×1, 5^2)
15×15 or $(15)^2$	=	225 (1×2, 5^2)
25×25 or $(25)^2$	=	625 (2×3, 5^2)
35×35 or $(35)^2$	=	1225 (3×4, 5^2)
45×45 or $(45)^2$	=	2025 (4×5, 5^2)
55×55 or $(55)^2$	=	3025 (5×6, 5^2)
65×65 or $(65)^2$	=	4225 (6×7, 5^2)
75×75 or $(75)^2$	=	5625 (7×8, 5^2)
85×85 or $(85)^2$	=	7225 (8×9, 5^2)
95×95 or $(95)^2$	=	9025 (9×10, 5^2)
105×105 or $(105)^2$	=	11025 (10×11, 5^2)
115×115 or $(115)^2$	=	13225 (11×12, 5^2)
125×125 or $(125)^2$	=	15625 (12×13, 5^2)

Number 8

Do you know that the number 8 is a great number to play with? Let us see the pattern given below:

1	×	8	+	1	=	9
12	×	8	+	2	=	98
123	×	8	+	3	=	987
1234	×	8	+	4	=	9876
12345	×	8	+	5	=	98765
123456	×	8	+	6	=	987654
1234567	×	8	+	7	=	9876543
12345678	×	8	+	8	=	98765432
123456789	×	8	+	9	=	987654321

Number 9

There are interesting results when some numbers are multiplied with the number 9. Look at the patterns when the sum is added to 1, 2, 3, …, 10.

0	×	9	+	1	=	1
1	×	9	+	2	=	11
12	×	9	+	3	=	111
123	×	9	+	4	=	1,111
1234	×	9	+	5	=	11,111
12345	×	9	+	6	=	1,11,111
123456	×	9	+	7	=	11,11,111
1234567	×	9	+	8	=	1,11,11,111
12345678	×	9	+	9	=	11,11,11,111
123456789	×	9	+	10	=	1,11,11,11,111

Some other beautiful patterns related to the number 9.

9	×	9	+	7	=	88
98	×	9	+	6	=	888
987	×	9	+	5	=	8,888
9876	×	9	+	4	=	88,888
98765	×	9	+	3	=	8,88,888
987654	×	9	+	2	=	88,88,888
9876543	×	9	+	1	=	8,88,88,888
98765432	×	9	+	0	=	88,88,88,888

Number 37

Look at the interesting results we get if we multiply 37 with the multiples of 3:

3	×	37	=	111
6	×	37	=	222
9	×	37	=	333
12	×	37	=	444
15	×	37	=	555
18	×	37	=	666
21	×	37	=	777
24	×	37	=	888
27	×	37	=	999

Some interesting reverse results:

	(+)			Sum	Product			(×)	
9	+	9	=	18	81	=	9	×	9
24	+	3	=	27	72	=	24	×	3
47	+	2	=	49	94	=	47	×	2
497	+	2	=	499	994	=	497	×	2

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